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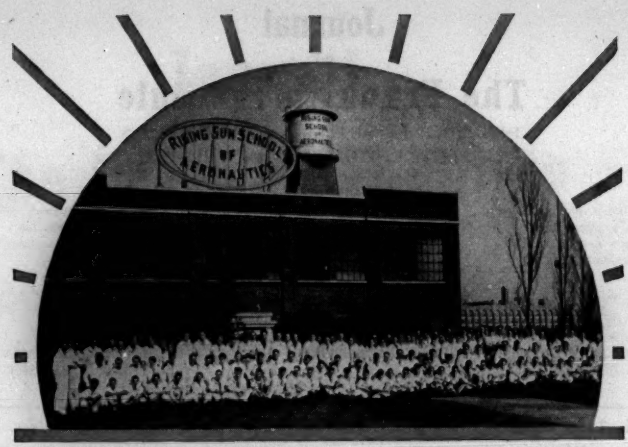

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Vol. 243 JANUARY, 1947 No. 1

ON METASTABLE STATES OF EQUILIBRIUM IN THERMODYNAMICS.

BY

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ABSTRACT.

In this paper the concept of reversibility is considered, and it is found that the question of reversibility of a process depends upon the number of independent variables of a system. For a single homogeneous closed phase subject only to mechanical and thermal changes, just two independent variables have been considered in classic thermodynamics. This practice requires that some states be characterized as metastable. Justification is offered for a change in this accepted view. Instead of classifying systems in phases, it is proposed to consider systems on the basis of number of types of transformation. A single gaseous phase subject to dissociation is found to be analogous to a liquid vapor system; each system has three independent variables, such as p, v, T, and equations of state will be of the form $\epsilon = F(p, v, T) = G(p, T, x)$ and $v = \Phi(p, T, x)$ where ϵ is the internal energy and x the degree of transformation. These equations include all metastable states.

1. INTRODUCTION.

In several previous papers (1) the writer has considered the possibility and the justification from a phenomenological point of view of treating metastable states of equilibrium on the same basis as those states which are normally included in an equation of state. The purpose of the present paper is to present some new material in support of this view. We shall limit the discussion to systems subject to mechanical and thermal changes (i.e., changes in pressure, volume, temperature), excluding effects due to gravitation, electric and magnetic forces, etc.

2. METASTABLE STATES FROM THE CLASSIC POINT OF VIEW.

Metastable states of equilibrium may be defined from the phenomenological point of view, and also on the basis of concepts fundamental

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to thermodynamics. From the former point of view, we may define a system as being metastable, if those parameters (p, V, T) used to define the state are constant with respect to time, *but which may at any instant exhibit a spontaneous change* (observed as an explosion in a gaseous phase, or rapid solidification from a supercooled liquid, etc., with accompanying changes in p, V, T and a liberation of heat). Actually, such changes do not exhibit true discontinuities, for in the laboratory it is possible to follow the progressive changes in p, V, T for even the most violent reactions.

Some metastable states are less stable than others. For example, a gaseous mixture of H_2 and O_2 is considered very unstable, although if chemically pure H_2 and O_2 are mixed in a closed vessel the system may persist for years without reacting. On the other hand, a mixture of N_2 and H_2 requires special care and the assistance of catalysts to make it react; under the best of conditions the reaction does not proceed with explosive violence. Also, in the liquid state we find supercooled water to be far less stable than supercooled quartz (fused silica). Experimentally, then, we find systems with very different degrees of stability; under certain conditions (usually the requirements are that the ingredients be of high chemical purity) practically all metastable states may be made to persist indefinitely.

The distinction between metastability and stability as considered in thermodynamics rests on the assumption that processes involving the latter states are reversible, whereas those involving the former states are not always reversible. The concept of reversibility is fundamental to thermodynamics, and is based on analogies to mechanical models. It is assumed that in order for a process to be reversible, it must be possible to carry this process forward and backward with infinitesimal velocity, or in other words, the process must be quasistatic. The net result of a closed cyclic process, if it is carried out reversibly, must be no change in the internal energy nor in all variables which determine the energy of the system and its surroundings. *However, before we can consider the reversibility of a process, we must establish the independent variables of the system. After settling this point, we may proceed to form a Carnot's cycle, or some other closed reversible cycle, and apply the law of conservation of energy and Clausius' or Kelvin's principle to develop the conditions for the thermodynamic equilibrium of a system. These latter conditions will necessarily be limited to states defined by the original set of independent variables, or its equivalent. Should we later discover that our system has more independent variables than originally contemplated, we would have to re-examine the criteria for equilibrium.*

Bearing this in mind, let us examine, for the sake of example, the treatment of Carnot's cycle. For the moment consider our system to be a single gaseous phase: this to be a real gas—not a theoretical one

which obeys the perfect gas law.* We carry this gas through the usual process involving two isothermals and two adiabatics in the (p, V)-plane. The assumption that is implied in drawing this closed cycle is that, for a given value of (p, V), the internal energy E, the temperature T, and all other variables of the system are completely defined. If this were not so—if, for example, we actually needed three instead of two independent variables to define the internal energy—each point on Carnot's cycle would represent many energy states, and in closing the cycle we would not necessarily bring our system back to its original energy content.

The most exact formulation for the criteria of thermodynamic equilibrium is obtained from a combination of the mathematical statements of the 1st and 2nd laws. For an isolated system, where the internal energy is constant, the necessary and sufficient condition is given by Gibbs (2) as $(\delta S)_E \leqq 0$, where S is the entropy of the system. If S is constant, the condition is $(\delta E)_S \geqq 0$. In his development of the proof of these conditions, Gibbs has explicitly left out those states which are subject to "passive forces," or what we would refer to at present as metastable states. In his development of the phase rule, Gibbs has explicitly assumed that a homogeneous system of fixed mass has only two degrees of freedom, i.e., (p, T) or (S, V) or (E, S), etc. The fact that metastable states have been left out of consideration in the development of the criteria for equllibrium, makes Gibbs' conditions necessary, but not sufficient in the sense that they tell us nothing about metastable states. Of course, the reader may argue that changes involving metastable states are not always reversible. This may be true if we judge reversibility only on the hypothesis that a system of a homogeneous phase of fixed mass has but two degrees of freedom, and that a complete reversal of a process involving this system is accomplished only by reversing its two independent variables. If, however, the system actually had a third independent degree of freedom, we would have to see whether processes involving metastable states were reversible when all three variables were varied infinitesimally.

For a chemical reaction within a homogeneous phase, the law of mass-action rests on the concept of reversibility, and ceases to hold when metastable states arise. Here, again, can we say that processes involving metastable states are not always reversible because this law does not apply; or, might we be justified in saying that the mass-action law, if based on the concept that a system of a homogeneous phase has only two degrees of freedom, is not entirely correct if our system actually has three or more degrees of freedom?

The problem, then, before us is—how many degrees of freedom does a thermodynamic system actually have, and if it has more than the

* In using the perfect gas law, we commit outselves at the very outset to two independent variables.

usually assigned number, how may metastable states be incorporated into our equation of state so as to enlarge the scope of this equation?

3. CONCEPTS OF CHANGE OF PHASE AND TRANSFORMATION.

The classification of systems into phases is quite natural, because most material systems appear to us in distinct phases. The classification is, however, not free from complications, due for example to the existence of glassy systems. In the change of matter from one phase to another, we have a distinct type of transformation, so that transformation and change of phase are synonymous in this sense. However, the former is much broader than the latter, for we have many transformations where there is no change of phase, as for example, dissociation within a single gaseous phase. A transformation would be any change (chemical or physical) which involves a latent heat of absorption or evolution.

We shall find, for our present purposes, that it will be more feasible to center our attention on the concept of transformation, bearing in mind that it also includes change of phase. We shall restrict outselves to a discussion of a closed system (fixed mass) which has a single type of transformation. The system may consist of liquid in equilibrium with its vapor, in which case the transformation will consist of evaporation (or condensation). Or, the system may consist of a single gaseous phase, where the transformation will be dissociation (or association).

4. AN ENLARGED EQUATION OF STATE.

We wish to enlarge our equation of state so as to include metastable states, and this will be done by increasing the number of independent variables. In selecting these variables we may adopt an operational (3) attitude and require that the variables form a minimum set in terms of which every state (including the metastable states) of the system is expressible.

Let us first consider the liquid-vapor system. At a given temperature T we may draw an isothermal in the (p, V)-plane, shown in Fig. 1 by the heavy line $FBCG$. BC is an isothermal-isopiestic, and points on this line represent states which are distinguished by different degrees of transformation x. We shall define x as the ratio, by weight, of the transformed state (vapor) to the total weight of the system. At B, $x = 0$, and the system will consist entirely of saturated vapor. CG represents states within a single gaseous phase, in which the transformation consists of dissociation, and is outside the type of transformation we wish to consider at present (i.e., liquid to vapor). Also, FB represents states within a liquid phase in which the transformation consists of liquid to solid, and is also outside our present consideration.

We know from experience that we can arbitrarily change x by infinitesimal amounts, keeping (p, T) = constant. To do so we add or

remove heat from the system, and the internal energy of the system will change by infinitesimal steps. We also know from experience that for such a system at state (p, T, x) we may add heat in infinitesimal quantities and observe no change in x, but actually observe a change in p, or T, or both. The system will be assuming a metastable state. With this change in p or T we expect to have an accompanying change in the internal energy of the system, even though x is constant.

Superheated and supercooled vapors and liquids are actually observed, and they may be considered as a persistence of form, described by the constancy of x. Consider our system at A, Fig. 1, where $x = x'$. If we hold x' and T constant, we may conceive of a process where p is increased by infinitesimal steps and V decreased correspondingly. The

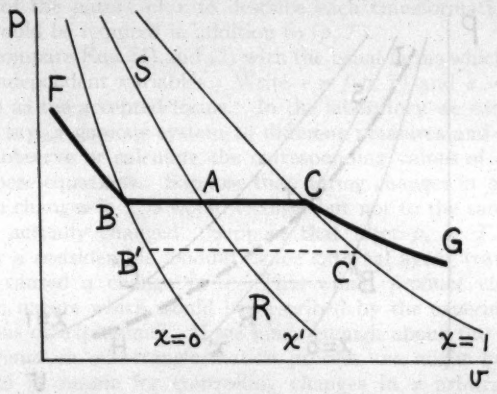

FIG. 1.

vapor and liquid phases would each be subjected to infinitesimal changes in (p, V), but no transformation would occur. The resulting states would be metastable. In theory, at least, we could develop the isothermal RAS, in which (x, T) are held constant and where the internal energy would be a function of (p, V). Suppose we define terms as follows: $\epsilon = E/M$, $v = V/M$, where E is the total internal energy, V the total volume, and M the total mass of the system. We see then that our liquid-vapor system of fixed mass has three independent variables (p, T, x) or (p, T, v), and the writer has suggested (1) equations of state of the form

$$\epsilon = F(p, T, v) = G(p, T, x), \tag{1}$$

from which may be obtained

$$v = \Phi(p, T, x). \tag{2}$$

These equations, it will be noted, include the metastable as well as the stable states; in fact, they are all treated on the same basis, at least as far as the equation of state is concerned. We may develop a family of isothermals at T = constant, shown in Fig. 1 by the light solid lines, where each isothermal will cross BC at a particular value of x. BC has been considered as a unique line in the two-dimensional system; however, this line is but one of many isothermal-isopiestics which we may draw at the temperature T, and the dashed line $B'C'$ will be another such line. At a different temperature we will have a different family of isothermals for the same transformational system.

Let us now consider a transformation within a single gaseous phase, for example $2NH_3 \rightleftharpoons N_2 + 3H_2$. At a given temperature T we may

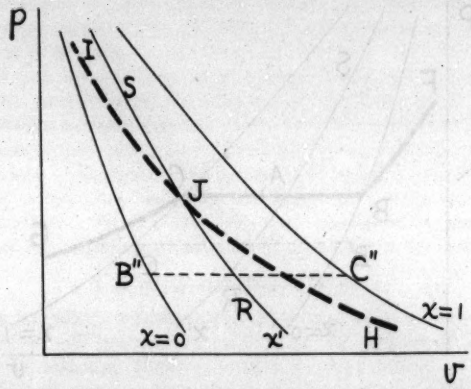

FIG. 2.

draw an isothermal in the (p, V) plane, shown in Fig. 2 by the heavy dashed line HI. This is the usual isothermal which satisfies the mass-action law. If we define x in a manner similar to above, as being the ratio by weight of the transformed state $(N_2 + 3H_2)$ to the total weight of the system, each point of HI will be distinguished by a different value of x, and the limits will be $x = 0$ and $x = 1$. Consider the state at J, where $x = x'$. Suppose that our system is free from any agent (catalytic or otherwise) which might cause a change in x', so that x' is held constant; also T is held constant. We find experimentally that we can, under these conditions, increase p by infinitesimal steps and obtain corresponding infinitesimal changes in V, and we may develop a line such as RJS where (T, x) are constant and (p, V) change. To each point on RJS there will correspond a certain internal energy. All points on RJS, except J, will be metastable. We may develop similar

curves at different values of x, all at the same temperature T, shown as light solid lines in Fig. 2; this family will be bounded by the lines $x = 0$ and $x = 1$. In theory, at least, we may follow a line such as $B''C''$, Fig. 2, an isothermal-isopiestic which is completely analogous to BC and $B'C'$ of Fig. 1. Comparing the two figures, we find that the two systems are entirely analogous in all points which are of any significance in thermodynamics. For the gaseous system we may use ϵ, v, x just as we have defined them for the liquid-vapor system, and Eqs. (1) and (2) will apply here.

The limitations of these equations must be borne in mind. If, instead of a single type of transformation, the system actually had several types taking place simultaneously, we would have to employ a variable of the nature of x to describe each transformation; these variables would be required in addition to (p, T).

Let us compare Eqs. (1) and (2) with the usual forms which contain only two independent variables. Write $\epsilon = f(p, T)$ and $x = g(p, T)$, $v = h(p, T)$ as the accepted forms. In the laboratory we can proceed to subject, say, a gaseous system to different pressures and temperatures, and observe or calculate the corresponding values of ϵ, x, v as based on these equations. Suppose that during changes in p, T there occurred no changes in x; v would change, but not to the same degree as when x actually changed. Suppose that after p, or T, or both changed by a considerable amount, some external agent (catalytic or otherwise) caused a change in x. This would produce changes in p, T, v of a nature which would be described by the experimenter as "spontaneous or explosive." If we knew enough about the effects of external agents on our transformation process, we might be able to utilize them as means for controlling changes in x arbitrarily. In other words, with this knowledge, x becomes an independent variable of our system. For liquid-vapor systems, changes in x under conditions of p, T = constant can be readily realized. So can changes in x be realized in a gaseous reaction, where the reaction responds to the presence of a specific catalyzer and which ceases when the catalyzer is removed. The question as to how x may be controlled in all systems is still open. In the study of reacting gaseous systems, the usual experimental procedure is to allow p, T to change simultaneously, and to observe x after the system has again reached equilibrium. The heat evolved in the reaction may, in itself, act as an accelerator simply by raising the temperature in the immediate neighborhood of the reacting components. If it were possible to devise experimental means for removing the heat as fast as it is generated, and thus to prevent local rises of temperature, we might realize conditions similar to ordinary evaporation at p, T = constant.

5. SOME FUNDAMENTAL RELATIONS.

From physical considerations we may assume that a transformation can take place only when there is a change in x. Let the latent heat of transformation, per unit mass, be represented by λ. There will be three distinct λ's: (1) with p, T = constant, write $\lambda_{p,T}$; (2) with v, T = constant, write $\lambda_{v,T}$; and (3) with p, v = constant, write $\lambda_{p,v}$. We know that for evaporation at p, T = constant, $\lambda_{p,T}$ is independent of x. Let us assume all λ's to be independent of x. If q is heat added to the system per unit mass, we may write

$$\lambda_{p,T} = \left(\frac{\partial q}{\partial x}\right)_{p,T} = f_1(p, T), \qquad \lambda_{v,T} = \left(\frac{\partial q}{\partial x}\right)_{v,T} = f_2(v, T),$$
$$\lambda_{p,v} = \left(\frac{\partial q}{\partial x}\right)_{p,v} = f_3(p, v), \tag{3}$$

where f_1, f_2, and f_3 are assumed to be single-valued functions of their respective two independent variables.

When x is held constant and T allowed to change, we may follow the practice in thermodynamics and define the following specific heats: (1) with x, p = constant, write $\kappa_{x,p}$; (2) with x, v = constant, write $\kappa_{x,v}$; and by definition

$$\kappa_{x,p} = \left(\frac{\partial q}{\partial T}\right)_{x,p} = g_1(p, T, x), \qquad \kappa_{x,v} = \left(\frac{\partial q}{\partial T}\right)_{x,v} = g_2(v, T, x), \tag{4}$$

where g_1, g_2 are in general assumed to be single-valued functions of three independent variables.

Differentiating the second of Eq. (1), we have

$$\begin{aligned} d\epsilon &= \left(\frac{\partial \epsilon}{\partial p}\right)_{T,x} dp + \left(\frac{\partial \epsilon}{\partial T}\right)_{x,p} dT + \left(\frac{\partial \epsilon}{\partial x}\right)_{p,T} dx \\ &= \left(\frac{\partial \epsilon}{\partial p}\right)_{T,x} dp + \left[\kappa_{x,p} - p\left(\frac{\partial v}{\partial T}\right)_{x,p}\right] dT \\ &\quad + \left[\lambda_{p,T} - p\left(\frac{\partial v}{\partial x}\right)_{p,T}\right] dx. \end{aligned} \tag{5}$$

If we write the equation of state in the form $\epsilon = f(v, T, x)$, we have

$$\begin{aligned} d\epsilon &= \left(\frac{\partial \epsilon}{\partial v}\right)_{T,x} dv + \left(\frac{\partial \epsilon}{\partial T}\right)_{x,v} dT + \left(\frac{\partial \epsilon}{\partial x}\right)_{v,T} dx \\ &= \left(\frac{\partial \epsilon}{\partial v}\right)_{T,x} dv + \kappa_{x,v}\, dT + \lambda_{v,T}\, dx. \end{aligned} \tag{6}$$

Also, if we write $\epsilon = f(p, v, x)$, we have

$$d\epsilon = \left(\frac{\partial \epsilon}{\partial p}\right)_{v,x} dp + \left(\frac{\partial \epsilon}{\partial v}\right)_{x,p} dv + \lambda_{p,v}\, dx. \tag{7}$$

Assuming (5), (6), and (7) to be exact differentials, we may develop the usual relations among the second partial derivatives. The following relations are of interest:

$$\left(\frac{\partial \lambda_{p,T}}{\partial T}\right)_{p,x} = \left(\frac{\partial \kappa_{x,p}}{\partial x}\right)_{p,T}, \tag{8}$$

$$\left(\frac{\partial \lambda_{v,T}}{\partial T}\right)_{v,x} = \left(\frac{\partial \kappa_{x,v}}{\partial x}\right)_{v,T}. \tag{9}$$

The internal energy ϵ is required to form an exact differential in our enlarged space on the basis of the first law. The following also have exact differentials in the (p, v, T) space * and may be considered as thermodynamic potential functions:

$$\Phi_\alpha = pvT, \tag{10}$$

$$H_\alpha = \epsilon + pv, \tag{11}$$

$$\Omega_\alpha = \frac{\epsilon + pv}{T}. \tag{12}$$

For the (p, T, x) space, the following functions will have exact differentials and therefore be potential functions:

$$\Phi_\beta = pxT, \tag{13}$$

$$H_\beta = \epsilon + px, \tag{14}$$

$$\Omega_\beta = \frac{\epsilon + px}{T}. \tag{15}$$

The α and β sets may be related by an equation between v and x. In the case of a liquid-vapor system, let v_1 and v_2 be the volumes, per unit mass, when $x = 0$ and $x = 1$, respectively. v_1 and v_2 will be single-valued functions of (p, T) only. Then, for a given value of (p, T)

$$v = v_1 + x(v_2 - v_1). \tag{16}$$

This relation may possibly hold also for a single gaseous system, and it would be worth while checking this experimentally.

Expressions for the work W done by, and the heat dq absorbed by

---

* If $F(p, v, T)$ represents one of these functions, then we may write

$$dF = (\partial F/\partial p)_{v,T}\, dp + (\partial F/\partial v)_{p,T}\, dv + (\partial F/\partial T)_{p,v}\, dT = P dp + Q dv + R dT$$

and the necessary and sufficient condition that dF be an exact differential is that

$$\left(\frac{\partial P}{\partial v}\right)_{T,p} = \left(\frac{\partial Q}{\partial p}\right)_{v,T}; \quad \left(\frac{\partial Q}{\partial T}\right)_{p,v} = \left(\frac{\partial R}{\partial v}\right)_{T,p}; \quad \left(\frac{\partial R}{\partial p}\right)_{v,T} = \left(\frac{\partial P}{\partial T}\right)_{p,v}.$$

The reader may apply these tests to Eqs. (10), (11) and (12).

the system may be derived as follows. From (10),

$$\begin{aligned} d\Phi_a &= pvdT + pTdv + vTdp \\ &= pvdT + TdW + vTdp, \end{aligned} \tag{17}$$

where $dW = pdv$. If (p, T) are held constant, we have on integration

$$W = \frac{1}{T}(\Phi_a'' - \Phi_a'), \tag{18}$$

where the prime and double-prime refer to the initial and final states of the system. If only T is held constant, we have

$$W = \frac{1}{T}(\Phi_a'' - \Phi_a') - \int_{(1)}^{(2)} vdp. \tag{19}$$

From (11) we have

$$\begin{aligned} dH_a &= d\epsilon + pdv + vdp \\ &= dq + vdp. \end{aligned} \tag{20}$$

If p = constant, we have

$$q = H_a'' - H_a'. \tag{21}$$

Differentiating (12), we have

$$d\Omega_a = -\frac{\Omega_a}{T}dT + \frac{dq}{T} + \frac{v}{T}dp. \tag{22}$$

If T = constant, we have

$$q = T(\Omega_a'' - \Omega_a') - \int_{(1)}^{(2)} vdp. \tag{23}$$

If p = constant, we have

$$q = T''\Omega_a'' - T'\Omega_a', \tag{24}$$

and if p, T = constant

$$q = T(\Omega_a'' - \Omega_a'). \tag{25}$$

Expressions for changes in x may be derived as follows: From (13),

$$d\Phi_\beta = pxdT + pTdx + Txdp. \tag{26}$$

If p, T = constant, we have

$$x'' - x' = \frac{1}{pT}(\Phi_\beta'' - \Phi_\beta'); \tag{27}$$

if p = constant, we have

$$x''T'' - x'T' = \frac{1}{p}(\Phi_\beta'' - \Phi_\beta'), \tag{28}$$

and if T = constant, we have

$$x''p'' - x'p' = \frac{1}{T}(\Phi_\beta'' - \Phi_\beta'). \tag{29}$$

6. CONCLUSIONS.

In this paper the writer has hoped to clarify certain notions which were not so well defined, nor represented in such rigorous fashion in his previous papers. The basic thought in the previous papers has been that from a phenomenological point of view all evidence seems to indicate that metastable states should not be excluded from our implicit form of equation of state. From the considerations of the present paper, it does seem that this practice has arisen because of the fact that processes involving metastable states have been assumed to be irreversible, and because irreversibility has been judged from a limited point of view.

The contention of the writer, and apparently a very simple truth when attention is directed to this point, is that before we can consider the question of reversibility and apply thermodynamics to a system, we must define the system. Mathematically, we must know the independent variables of the system. The writer is also of the opinion that in the behavior of systems, the important and basic concept is not change of phase, *but the much broader concept of a transformation of any type*, of which change of phase is a specific case. Transformations will include all chemical reactions within a single phase, dissociation, ionization, etc., as well as evaporation, crystallization, etc.

Realizing that unless we know and control all the independent variables of a system some of the states of the system will be unstable or metastable, the writer believes that in seeking for the additional variables, we will be able to arrive at a better understanding of "explosive and spontaneous" chemical and physical changes. Such changes, when they occur, may imply that certain variables of the system (existing, but possibly not known to us) have not been under control. If these variables actually change, we refer to the process as a "spontaneous" change; if the variables do not change, we may assume our system to be in a "metastable" state.

In the studies of the complex phenomena of catalysis from a thermodynamic standpoint, suppose we proceed on the assumption that when a catalyzer is introduced into a system, the original type of reaction is divided into stages to form intermediate compounds. The catalyzer has then essentially increased the number of types of transformation, and for each type we need an additional independent variable. It is therefore quite probable that some of these variables (possibly unknown to us, or at least not taken into account in the equation of state for the system) may change, and thus "catalyze" the original reacting system.

In section 5, above, the writer has mentioned certain characteristic functions which form exact differentials in the enlarged space. The entropy function has not been treated, but will be reserved for a later paper. In the paper which follows the present one, the writer utilizes

this new approach to thermodynamics to develop an explicit equation of state for a system with one type of transformation.

7. ACKNOWLEDGMENTS.

I wish to take this opportunity to express my deep appreciation to my friends, John M. Reiner of Washington University and Maurice H. Slud of Brown University, with whom I have discussed this paper in all its details, and who have contributed valuable suggestions and criticism.

Paper—"On Metastable States of Equilibrium in Thermodynamics."

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NATURAL MODES OF VIBRATION OF SIMPLE FRAMES.

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INTRODUCTION.

Although it is possible to obtain approximate natural frequencies for a frame by the application of Rayleigh's method, this method becomes rather difficult for the higher modes of vibration. The Rayleigh method consists essentially of the determination of the equivalent lumped parameters of a system with distributed parameters. This equivalence is established for a configuration of the system approximating that existing during actual vibration. The natural frequency is then determined for the equivalent lumped system.

To obtain an accurate solution for all of the modes of vibration, however, it is necessary to recognize the distributed nature of the parameters and to solve the appropriate differential equation. This type of analysis has been made for a simple beam (1, 4) and for a beam on many supports (2, 3, 4). It is the purpose of this paper to illustrate the application of this form of analysis to simple frames by considering the individual sides as simple beams with the proper end conditions. Solutions are derived for rectangular and triangular frames. Frequency curves and vibrational modes are illustrated for the first two modes of each type of vibration for rectangular and isosceles-triangular frames whose sides have the same distributed parameters. The equilateral case is considered in detail due to certain singularities which appear. Finally the applicability of the results to certain general types of frames is indicated.

DIFFERENTIAL EQUATION FOR A SIMPLE BEAM.

If we assume that the vibration occurs in one of the principal planes of flexure of the beam and that the cross sectional dimensions are small compared to the length, we can apply the following familiar form of the deflection curve:

$$\frac{d^2y}{dx^2} = -\frac{M}{EI}, \tag{1}$$

where M is the bending moment, E is the modulus of elasticity, and I is the moment of inertia of the beam section. This is based on a conventional coordinate system as indicated by the dotted lines in Fig. 1. The solid line represents the neutral axis of the beam. It will be noted

that one end of the beam is at the origin and the beam extends along the positive x-axis. The arrowhead is placed on the end of the beam away from the origin. In this manner just placing an arrowhead on a beam will automatically fix its system of reference axes. This system will be used later in the analysis of the frame.

Since we are dealing with uniform beams, and further since the first derivative of the bending moment with respect to x is equal to the shear-

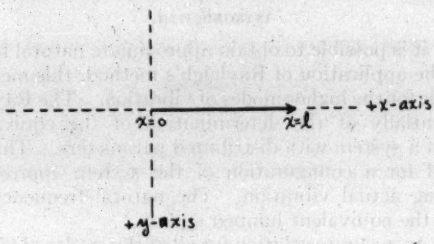


Fig. 1.

ing force Q, we get the following on differentiating Eq. (1) with respect to x:

$$\frac{d^3y}{dx^3} = -\frac{Q}{EI}. \tag{2}$$

If we should differentiate again, and remembering that minus the first derivative of the shearing force with respect to x is the load intensity w, we get:

$$\frac{d^4y}{dx^4} = \frac{w}{EI}. \tag{3}$$

During vibration the beam will be loaded by inertia forces so that we can replace w by the inertia force which gives:

$$\left.\begin{aligned} &\frac{\partial^4y}{\partial x^4} = -\frac{m}{EI}\frac{\partial^2y}{\partial t^2}, \\ &\frac{\partial^2y}{\partial t^2} + r^2\frac{\partial^4y}{\partial x^4} = 0; \qquad r^2 = \frac{EI}{m}, \end{aligned}\right\} \tag{4}$$

where m is the mass per unit length of the beam.

The general solution of Eq. (4) is:

$$y = X(K_1 \cos \omega t + K_2 \sin \omega t), \tag{5}$$

$$X = R_1 \sin kx + R_2 \cos kx + R_3 \sinh kx + R_4 \cosh kx,$$
$$k^2 = \frac{\omega}{r} = 2\pi f \sqrt{\frac{m}{EI}}. \qquad (6)$$

The function X of Eq. (6) gives the variation of amplitude of vibration along the beam at any instant. By applying the proper end conditions to the function X and its derivatives we can obtain equations for the natural frequencies of a simple beam.

RECTANGULAR FRAME.

Let us consider a disjointed rectangular frame as shown in Fig. 2 with the sides A, B, C, and D each of which satisfy the conditions assumed in the preceding development. Each side is given an arrowhead

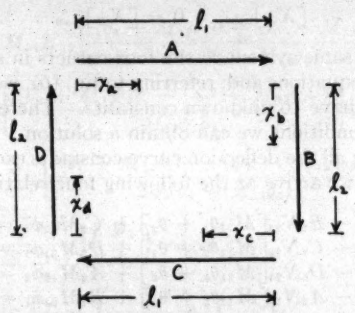


Fig. 2.

which fixes its own set of coordinate axes as shown in Fig. 1. In what follows capital letter subscripts are used to identify the X functions for each individual side while lower case letters as subscripts are used to identify the x coordinates. Further, in applying Eq. (6) the R constants become the A's, B's, etc. with the proper numerical subscripts.

It is assumed that the corners of the frame are rigidly tied together and pivoted, that is free to rotate but unable to have translatory motion. These assumptions immediately fix the end conditions imposed by the frame on the individual beams. The sides of length l_1 are assumed to have the parameters E_1, I_1, and m_1, namely the sides A and C, while the sides B and D are assumed to have the parameters E_2, I_2, and m_2 with the corresponding length l_2.

Considering the corner where beams A and B adjoin in Fig. 2, a rigid corner requires that the slope of beam A at $x_a = l_1$ be equal to the

slope of beam B at $x_b = 0$ or:

$$\left[\frac{dX_A}{dx_a}\right]_{x_a=l_1} = \left[\frac{dX_B}{dx_b}\right]_{x=0}, \tag{7}$$

and in addition that the bending moments at these points be equal, which gives by Eq. (1):

$$E_1I_1\left[\frac{d^2X_A}{dx_a^2}\right]_{x_a=l_1} = E_2I_2\left[\frac{d^2X_B}{dx_b^2}\right]_{x_b=0}. \tag{8}$$

The final end conditions are based on the requirement that the corners have no translatory motion, so that the displacement of beam A at $x_a = l_1$ is zero and the displacement of beam B at $x_b = 0$ is also zero:

$$[X_A]_{x_a=l_1} = 0 = [X_B]_{x_b=0}. \tag{9}$$

Applying this same system to the four corners in succession we obtain a total of 16 equations and, referring to Eq. (6), we see that for four sides we will also have 16 unknown constants. Therefore, by applying these boundary conditions we can obtain a solution.

By eliminating all the deflection curve constants except those having the manuscript 2 we arrive at the following four relations:

$$\left.\begin{aligned} A_2\phi_1 - B_2N_{21}[M_{21}\theta_1 + \theta_2] + C_2M_{12}\phi_2 &= 0,\\ B_2\phi_2 - C_2N_{12}[M_{12}\theta_2 + \theta_1] + D_2M_{21}\phi_1 &= 0,\\ C_2\phi_1 - D_2N_{21}[M_{21}\theta_1 + \theta_2] + A_2M_{12}\phi_2 &= 0,\\ D_2\phi_2 - A_2N_{12}[M_{12}\theta_2 + \theta_1] + B_2M_{21}\phi_1 &= 0, \end{aligned}\right\} \tag{10}$$

where:

$$\phi_1 = \operatorname{csch} k_1l_1 - \csc k_1l_1; \qquad \theta_1 = \coth k_1l_1 - \cot k_1l_1,$$

$$\phi_2 = \operatorname{csch} k_2l_2 - \csc k_2l_2; \qquad \theta_2 = \coth k_2l_2 - \cot k_2l_2,$$

$$M_{12} = \frac{1}{M_{21}} = \frac{E_1I_1k_1}{E_2I_2k_2}; \qquad N_{12} = \frac{1}{N_{21}} = \frac{k_1}{k_2}.$$

If we equate the determinant of the coefficients of these remaining constants to zero we obtain the characteristic frequency equation for the frame. This turns out to be:

$$\begin{aligned}&[(\phi_1 + \theta_1) + M_{12}(\phi_2 + \theta_2)]\\ &\quad\times[(\phi_1 - \theta_1) + M_{12}(\phi_2 - \theta_2)][(\phi_1 - \theta_1) - M_{12}(\phi_2 + \theta_2)]\\ &\qquad\times[(\phi_1 + \theta_1) - M_{12}(\phi_2 - \theta_2)] = 0.\end{aligned} \tag{11}$$

The relative amplitude of vibration along the sides of the frame for any particular mode may be found by taking the ratio of the X's to any one of the A's, B's, C's, or D's. In the following, A_2 has been taken as

reference. The general expressions for the deflection curves and the values of the coefficient ratios derived for the rectangular frame are:

$$\left.\begin{aligned}
\frac{X_A}{A_2} &= \frac{A_1}{A_2}\sin k_1x_a + (\cos k_1x_a - \cosh k_1x_a) + \frac{A_3}{A_2}\sinh k_1x_a,\\
\frac{X_B}{A_2} &= \frac{B_1}{A_2}\sin k_2x_b + \frac{B_2}{A_2}(\cos k_2x_b - \cosh k_2x_b) + \frac{B_3}{A_2}\sinh k_2x_b,\\
\frac{X_C}{A_2} &= \frac{C_1}{A_2}\sin k_1x_c + \frac{C_2}{A_2}(\cos k_1x_c - \cosh k_1x_c) + \frac{C_3}{A_2}\sinh k_1x_c,\\
\frac{X_D}{A_2} &= \frac{D_1}{A_2}\sin k_2x_d + \frac{D_2}{A_2}(\cos k_2x_d - \cosh k_2x_d) + \frac{D_3}{A_2}\sinh k_2x_d,
\end{aligned}\right\} \tag{12}$$

where:

$$\frac{B_2}{A_2} = M_{12}N_{12}\left[\frac{(\theta_1 + M_{12}\theta_2)^2 + \phi_1^2 - M_{12}^2\phi_2^2}{2\phi_1(\theta_1 + M_{12}\theta_2)}\right],$$

$$\frac{C_2}{A_2} = \left[\frac{(\theta_1 + M_{12}\theta_2)^2 - (\phi_1^2 + M_{12}^2\phi_2^2)}{2M_{12}\phi_1\phi_2}\right],$$

$$\frac{D_2}{A_2} = N_{12}\left[\frac{(\theta_1 + M_{12}\theta_2)^2 - (\phi_1^2 - M_{12}^2\phi_2^2)}{2\phi_2(\theta_1 + M_{12}\theta_2)}\right],$$

$$\frac{A_1}{A_2} = \left(\frac{\csc k_1l_1}{M_{12}N_{12}}\right)\left(\frac{B_2}{A_2}\right) - \cot k_1l_1,$$

$$\frac{A_3}{A_2} = \coth k_1l_1 - \left(\frac{\operatorname{csch} k_1l_1}{M_{12}N_{12}}\right)\left(\frac{B_2}{A_2}\right),$$

$$\frac{B_1}{A_2} = (M_{12}N_{12}\csc k_2l_2)\left(\frac{C_2}{A_2}\right) - (\cot k_2l_2)\left(\frac{B_2}{A_2}\right),$$

$$\frac{B_3}{A_2} = (\coth k_2l_2)\left(\frac{B_2}{A_2}\right) - (M_{12}N_{12}\operatorname{csch} k_2l_2)\left(\frac{C_2}{A_2}\right),$$

$$\frac{C_1}{A_2} = \left(\frac{\csc k_1l_1}{M_{12}N_{12}}\right)\left(\frac{D_2}{A_2}\right) - (\cot k_1l_1)\left(\frac{C_2}{A_2}\right),$$

$$\frac{C_3}{A_2} = (\coth k_1l_1)\left(\frac{C_2}{A_2}\right) - \left(\frac{\operatorname{csch} k_1l_1}{M_{12}N_{12}}\right)\left(\frac{D_2}{A_2}\right),$$

$$\frac{D_1}{A_2} = M_{12}N_{12}\csc k_2l_2 - (\cot k_2l_2)\left(\frac{D_2}{A_2}\right),$$

$$\frac{D_3}{A_2} = (\coth k_2l_2)\left(\frac{D_2}{A_2}\right) - M_{12}N_{12}\operatorname{csch} k_2l_2.$$

TYPES OF VIBRATION.

If we consider the general frequency again; that is, Eq. (11), we can see that it contains four independent factors. Each one of these factors determines a special type of vibration as will be shown.

Let us return to our disjointed frame in Fig. 2. If in addition to the end conditions already imposed, we formulate additional ones, we can obtain special types of vibration which will still be included in the general solution since we have retained all the original boundary conditions.

Restraint (A).

First let us require that the two ends of any side have identical slopes. When the equations are solved with these additional boundary conditions we obtain the following frequency equation:

$$[(\phi_1 + \theta_1) + M_{12}(\phi_2 + \theta_2)] = 0, \tag{13}$$

which we recognize to be the first factor in the general frequency equation. The deflection constant ratios for use in Eq. (12) reduce to:

$$\left.\begin{aligned}
&\frac{C_2}{A_2} = 1, \qquad \frac{B_2}{A_2} = \frac{D_2}{A_2} = -M_{12}N_{12},\\
&\frac{A_1}{A_2} = \frac{C_1}{A_2} = -(\csc k_1 l_1 + \cot k_1 l_1),\\
&\frac{A_3}{A_2} = \frac{C_3}{A_2} = \coth k_1 l_1 + \operatorname{csch} k_1 l_1,\\
&\frac{B_1}{A_2} = \frac{D_1}{A_2} = M_{12}N_{12}(\csc k_2 l_2 + \cot k_2 l_2),\\
&\frac{B_3}{A_2} = \frac{D_3}{A_2} = -M_{12}N_{12}(\coth k_2 l_2 + \operatorname{csch} k_2 l_2),
\end{aligned}\right\} \tag{14}$$

where:

$$M_{12} = \sqrt[4]{\left(\frac{E_1 I_1}{E_2 I_2}\right)\left(\frac{m_1}{m_2}\right)}, \qquad N_{12} = \sqrt[4]{\frac{E_2 I_2 m_1}{E_1 I_1 m_2}}, \qquad M_{12}N_{12} = \sqrt{\frac{E_1 I_1 m_1}{E_2 I_2 m_2}}.$$

These values show that the deflection curves for the different sides are related as follows:

$$\left.\begin{aligned}
\frac{X_A}{A_2} &= \frac{X_C}{A_2},\\
\frac{X_B}{A_2} &= \frac{X_D}{A_2}.
\end{aligned}\right\} \tag{15}$$

The additional boundary conditions imposed for this type of vibration allow us to predict in a general way the relative slopes of the ends of each side. Fig. 3 indicates the end pattern of the sides produced by the additional restrictions of Restraint (A). The heavy dotted lines indicate the undeflected frame, the heavy curved portions indicate the required end pattern, and the light dotted curve illustrates the lowest mode of vibration conforming to this particular end pattern. Since these end patterns are relative in nature, the slope of side A at its origin is taken as positive in all end pattern figures for simplicity.

Restraint (B).

Next let us require that the two ends of any side have slopes that are the negative of each other. When the equations are solved with these additional boundary conditions we obtain the following frequency equation:

$$[(\phi_1 - \theta_1) + M_{12}(\phi_2 - \theta_2)] = 0, \tag{16}$$

which we recognize to be the second factor in Eq. (11). The deflection constant ratios for use in Eq. (12) become:

$$\left.\begin{aligned}
&\frac{C_2}{A_2} = 1, \qquad \frac{B_2}{A_2} = \frac{D_2}{A_2} = M_{12}N_{12},\\
&\frac{A_1}{A_2} = \frac{C_1}{A_2} = \csc k_1 l_1 - \cot k_1 l_1,\\
&\frac{A_3}{A_2} = \frac{C_3}{A_2} = \coth k_1 l_1 - \operatorname{csch} k_1 l_1,\\
&\frac{B_1}{A_2} = \frac{D_1}{A_2} = M_{12}N_{12}\,(\csc k_2 l_2 - \cot k_2 l_2),\\
&\frac{B_3}{A_2} = \frac{D_3}{A_2} = M_{12}N_{12}\,(\coth k_2 l_2 - \operatorname{csch} k_2 l_2).
\end{aligned}\right\} \tag{17}$$

These values show that the deflection curves of the different sides are related as follows:

$$\left.\begin{aligned}
\frac{X_A}{A_2} &= \frac{X_C}{A_2},\\
\frac{X_B}{A_2} &= \frac{X_D}{A_2}.
\end{aligned}\right\} \tag{18}$$

In Fig. 4 we have illustrated the end conditions produced by Restraint (B). As before the heavy dotted lines indicate the undeflected frame, the heavy curved portions indicate the required end pattern, and the light dotted curve illustrates the lowest mode of vibration conforming to this particular end pattern.

Restraint (C).

As our third set of additional conditions let us require that for each of the sides A and C the two ends have slopes that are the negative of each other, and for each of the sides B and D the two ends have identical slopes. When the equations are solved with these additional boundary conditions, we obtain the following frequency equation:

$$[(\phi_1 - \theta_1) - M_{12}(\phi_2 + \theta_2)] = 0, \tag{19}$$

which we recognize to be the third factor in the general frequency equation. The deflection constant ratios for use in Eq. (12) become:

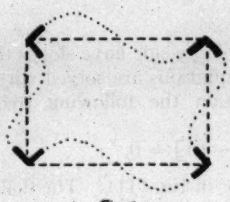


Fig. 3

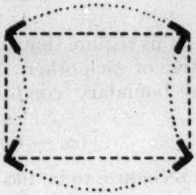


Fig. 4

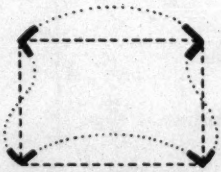


Fig. 5

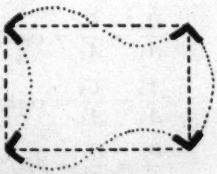


Fig. 6

$$\left.\begin{aligned}
&\frac{C_2}{A_2} = -1, \qquad \frac{B_2}{A_2} = -\frac{D_2}{A_2} = M_{12}N_{12},\\
&\frac{A_1}{A_2} = -\frac{C_1}{A_2} = (\csc k_1 l_1 - \cot k_1 l_1),\\
&\frac{A_3}{A_2} = -\frac{C_3}{A_2} = (\coth k_1 l_1 - \operatorname{csch} k_1 l_1),\\
&\frac{B_1}{A_2} = -\frac{D_1}{A_2} = -M_{12}N_{12}(\csc k_2 l_2 + \cot k_2 l_2),\\
&\frac{B_3}{A_2} = -\frac{D_3}{A_2} = M_{12}N_{12}(\coth k_2 l_2 + \operatorname{csch} k_2 l_2).
\end{aligned}\right\} \tag{20}$$

These values show that the deflection curves of the different sides are related as follows:

$$\left.\begin{aligned} \frac{X_A}{A_2} &= -\frac{X_C}{A_2}, \\ \frac{X_B}{A_2} &= -\frac{X_D}{A_2}. \end{aligned}\right\} \tag{12}$$

In Fig. 5 the end pattern for Restraint (C) is illustrated.

Restraint (D).

As our final set of additional conditions let us require that for each of the sides A and C the two ends have identical slopes, while for each of the sides B and D the two ends have slopes that are the negative of each other. When the equations are solved with these additional boundry conditions, we obtain the following frequency equation:

$$[(\phi_1 + \theta_1) - M_{12}(\phi_2 - \theta_2)] = 0, \tag{22}$$

which we recognize as the fourth factor in the general frequency equation, Eq. (11). The deflection constant ratios for use in Eq. (12) become:

$$\left.\begin{aligned} &\frac{C_2}{A_2} = -1, \qquad \frac{B_2}{A_2} = -\frac{D_2}{A_2} = -M_{12}N_{12}, \\ &\frac{A_1}{A_2} = -\frac{C_1}{A_2} = -(\csc k_1 l_1 + \cot k_1 l_1), \\ &\frac{A_3}{A_2} = -\frac{C_3}{A_2} = \coth k_1 l_1 + \operatorname{csch} k_1 l_1, \\ &\frac{B_1}{A_2} = -\frac{D_1}{A_2} = -M_{12}N_{12}(\csc k_2 l_2 - \cot k_2 l_2), \\ &\frac{B_3}{A_2} = -\frac{D_3}{A_2} = -M_{12}N_{12}(\coth k_2 l_2 - \operatorname{csch} k_2 l_2). \end{aligned}\right\} \tag{23}$$

These values show that for this type of vibration the deflection curves of the different sides are related as follows:

$$\left.\begin{aligned} \frac{X_A}{A_2} &= -\frac{X_C}{A_2}, \\ \frac{X_B}{A_2} &= -\frac{X_D}{A_2}. \end{aligned}\right\} \tag{24}$$

In Fig. 6 the end pattern for Restraint (D) is illustrated.

It should be noted from the results given in Eqs. (15), (18), (21) and (24) that it is only necessary to evaluate the deflection curves for two adjoining sides to obtain the complete deflection curve for the frame.

EVALUATION OF THE RESONANT FREQUENCIES.

The individual factors of the general frequency equation can be written as follows:

$$\left.\begin{aligned} &(a) \quad (\phi_1 + \theta_1) = -M_{12}(\phi_2 + \theta_2),\\ &(b) \quad (\phi_1 - \theta_1) = -M_{12}(\phi_2 - \theta_2),\\ &(c) \quad (\phi_1 - \theta_1) = M_{12}(\phi_2 + \theta_2),\\ &(d) \quad (\phi_1 + \theta_1) = M_{12}(\phi_2 - \theta_2), \end{aligned}\right\} \tag{25}$$

where:

$$\begin{aligned} \phi_1 &= \operatorname{csch} k_1 l_1 - \csc k_1 l_1,\\ \phi_2 &= \operatorname{csch} k_2 l_2 - \csc k_2 l_2,\\ \theta_1 &= \coth k_1 l_1 - \cot k_1 l_1,\\ \theta_2 &= \coth k_2 l_2 - \cot k_2 l_2,\\ M_{12} &= \sqrt[4]{\left(\frac{E_1 I_1}{E_2 I_2}\right)^3 \left(\frac{m_1}{m_2}\right)}. \end{aligned}$$

Eq. (25) really only involves two distinct functions and these are:

$$\begin{aligned} (\phi + \theta) &= \operatorname{csch} z + \coth z - \csc z - \cot z,\\ (\phi - \theta) &= \operatorname{csch} z - \coth z - \csc z + \cot z. \end{aligned} \tag{26}$$

These functions can be plotted very simply from values given in tables of trigonometric and hyperbolic functions. From the graphs of these two functions it is possible to pick off pairs of values of the parameter z satisfying any one of the relations in Eq. (25), which values will be $k_1 l_1$ and $k_2 l_2$. If we define n to be the ratio of $k_1 l_1$ over $k_2 l_2$ then we can plot curves of $k_2 l_1$ against the ratio n for the different modes. If for simplicity we let K_2 be equal to $k_2 l_2$ the frequency of each mode of vibration from Eq. (6) will be given by:

$$f = \frac{K_2^2}{2\pi l_2^2}\sqrt{\frac{E_2 I_2}{m_2}}. \tag{27}$$

The actual form of the deflection curves for any particular resonant frequency can be plotted from Eq. (12), using Eq. (14), (17), (20), or (23), depending on the particular factor of the general frequency equation involved. As has been pointed out, opposite sides of the frame will have the same or exactly opposite deflection curves so that it is only necessary to plot the deflection curve for two adjoining sides to determine the complete deflection curve of the frame.

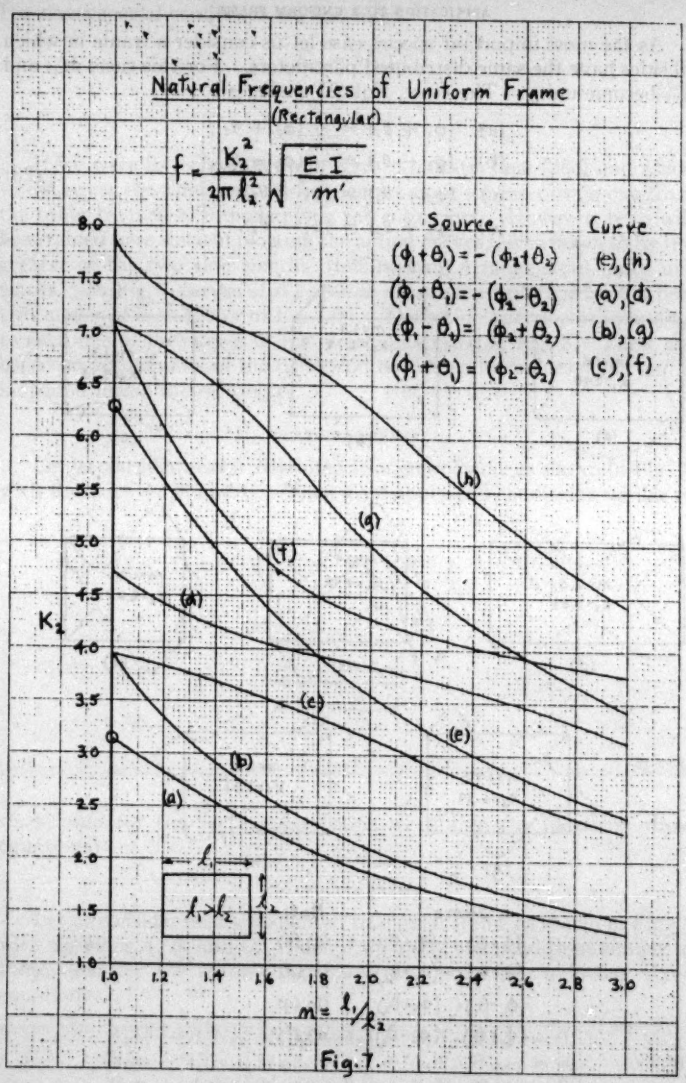
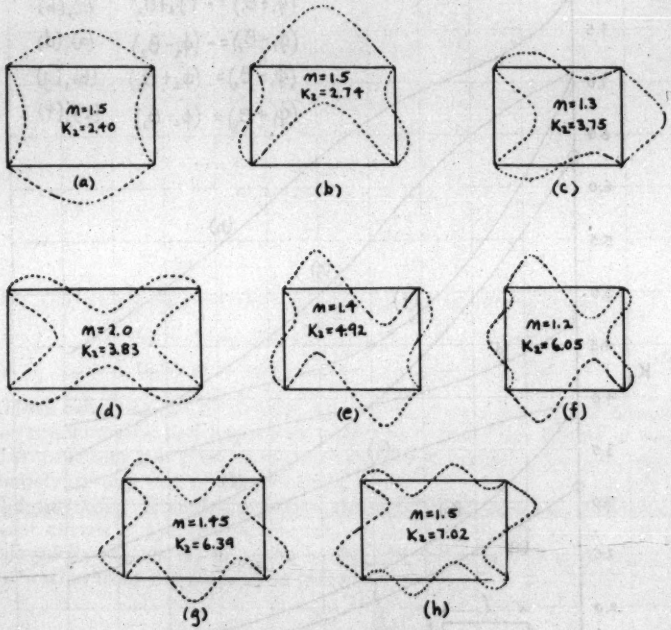

Natural Frequencies of Uniform Frame
(Rectangular)
$f = \frac{K_2^2}{2\pi l_2^2}\sqrt{\frac{EI}{m'}}$
Source Curve
$(\phi_1+\theta_1) = -(\phi_2+\theta_2)$ (e), (h)
$(\phi_1-\theta_1) = -(\phi_2-\theta_2)$ (a), (d)
$(\phi_1-\theta_1) = (\phi_2+\theta_2)$ (b), (g)
$(\phi_1+\theta_1) = (\phi_2-\theta_2)$ (c), (f)
K_2
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8 3.0
$m = l_1/l_2$
(a) (b) (c) (d) (e) (f) (g) (h)
l_1
l_2
$l_1 = l_2$

Fig. 7.

APPLICATION TO A UNIFORM FRAME.

As the most important special case let us consider a frame in which all sides have the same distributed parameters. For this case, M_{12} and N_{12} become unity. Therefore, Eq. (25) reduces to:

$$\left.\begin{aligned} &\text{(a)} \quad (\phi_1 + \theta_1) = -(\phi_2 + \theta_2), \\ &\text{(b)} \quad (\phi_1 - \theta_1) = -(\phi_2 - \theta_2), \\ &\text{(c)} \quad (\phi_1 - \theta_1) = (\phi_2 + \theta_2), \\ &\text{(d)} \quad (\phi_1 + \theta_1) = (\phi_2 - \theta_2). \end{aligned}\right\} \tag{28}$$



Source	Mode
$(\phi_1+\theta_1)=-(\phi_2+\theta_2)$	(c),(h)
$(\phi_1-\theta_1)=-(\phi_2-\theta_2)$	(a),(d)
$(\phi_1-\theta_1)=(\phi_2+\theta_2)$	(b),(g)
$(\phi_1+\theta_1)=(\phi_2-\theta_2)$	(c),(f)

Fig. 8.

For these special conditions we also have:

$$\left.\begin{aligned} K_2 &= kl_2, \\ n &= \frac{l_1}{l_2}. \end{aligned}\right\} \tag{29}$$

If we were to let n vary from zero to infinity Eqs. (28c) and (28d) would cover the same natural frequencies since we can obtain one from the other by merely interchanging the subscripts. However, both will be retained here since in plotting the actual curves l_1 was taken to be the greater of the two side lengths, thus making n range from unity upwards. In Fig. 7 curves of K_2 against n have been plotted for the first two modes of each factor of Eq. (28). Deflection curves corresponding to each one of the curves in Fig. 7 are illustrated in Fig. 8. These are based on actual plots of the deflection equations for values of K_2 and n indicated inside the rectangles.

TRIANGULAR FRAME.

Let us consider next a disjointed triangular frame as shown in Fig. 9, with the sides A, B, and C. Then if we apply end conditions similar to

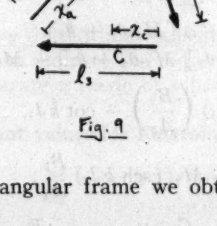


Fig. 9

those used for the rectangular frame we obtain the following three equations:

TRIANGULAR FRAME.

Let us consider next a disjointed triangular frame as shown in Fig. 9, with the sides A, B, and C. Then if we apply end conditions similar to those used for the rectangular frame we obtain the following three equations:

$$\left.\begin{aligned} A_2\phi_1 - B_2N_{21}[M_{21}\theta_1 + \theta_2] + C_2N_{31}M_{32}\phi_2 &= 0, \\ B_2\phi_2 - C_2N_{32}[M_{32}\theta_2 + \theta_3] + A_2N_{12}M_{13}\phi_3 &= 0, \\ C_2\phi_3 - A_2N_{13}[M_{13}\theta_3 + \theta_1] + B_2N_{23}M_{21}\phi_1 &= 0, \end{aligned}\right\} \tag{30}$$

where:

$$\phi_n = \operatorname{csch} k_n l_n - \csc k_n l_n, \qquad \theta_n = \coth k_n l_n - \cot k_n l_n,$$

$$M_{nm} = \frac{E_n I_n k_n}{E_m I_m k_m}, \qquad N_{nm} = \frac{k_n}{k_m}.$$

The subscripts 1, 2, and 3 refer to sides A, B, and C respectively.

These equations yield the following general frequency equation for a triangular frame:

$$\begin{aligned} 2\phi_1\phi_2\phi_3 &+ M_{21}\phi_1^2[M_{32}\theta_2 + \theta_3] \\ &+ M_{32}\phi_2^2[M_{13}\theta_3 + \theta_1] + M_{13}\phi_3^2[M_{21}\theta_1 + \theta_2] \\ &- [M_{21}\theta_1 + \theta_2][M_{32}\theta_2 + \theta_3][M_{13}\theta_3 + \theta_1] = 0. \end{aligned} \tag{31}$$

The general forms of the deflection curve equations and constants are:

$$\left.\begin{aligned}
\frac{X_A}{A_2} &= \frac{A_1}{A_2}\sin k_1x_a + (\cos k_1x_a - \cosh k_1x_a) + \frac{A_3}{A_2}\sinh k_1x_a, \\
\frac{X_B}{A_2} &= \frac{B_1}{A_2}\sin k_2x_b + \frac{B_2}{A_2}(\cos k_2x_b - \cosh k_2x_b) + \frac{B_3}{A_2}\sinh k_2x_b, \\
\frac{X_C}{A_2} &= \frac{C_1}{A_2}\sin k_3x_c + \frac{C_2}{A_2}(\cos k_3x_c - \cosh k_3x_c) + \frac{C_3}{A_2}\sinh k_3x_c, \\
\frac{C_2}{A_2} &= \frac{\phi_1\phi_2 + M_{13}\phi_3[M_{21}\theta_1 + \theta_2]}{N_{31}\{[M_{21}\theta_1 + \theta_2][M_{32}\theta_2 + \theta_3] - M_{32}\phi_2^2\}}, \\
\frac{B_2}{A_2} &= \frac{M_{12}\phi_2\phi_3 + \phi_1[M_{32}\theta_2 + \theta_3]}{N_{21}\{[M_{21}\theta_1 + \theta_2][M_{32}\theta_2 + \theta_3] - M_{32}\phi_2^2\}}, \\
\frac{A_1}{A_2} &= (N_{21}M_{21}\csc k_1l_1)\left(\frac{B_2}{A_2}\right) - \cot k_1l_1, \\
\frac{A_3}{A_2} &= \coth k_1l_1 - (N_{21}M_{21}\operatorname{csch} k_1l_1)\frac{B_2}{A_2}, \\
\frac{B_1}{A_2} &= (N_{32}M_{32}\csc k_2l_2)\frac{C_2}{A_2} - (\cot k_2l_2)\frac{B_2}{A_2}, \\
\frac{B_3}{A_2} &= (\coth k_2l_2)\frac{B_2}{A_2} - (N_{32}M_{32}\operatorname{csch} k_2l_2)\frac{C_2}{A_2}, \\
\frac{C_1}{A_2} &= N_{13}M_{13}\csc k_3l_3 - (\cot k_3l_3)\frac{C_2}{A_2}, \\
\frac{C_3}{A_2} &= (\coth k_3l_3)\frac{C_2}{A_2} - N_{13}M_{13}\operatorname{csch} k_3l_3.
\end{aligned}\right\} \tag{32}$$

ISOSCELES TRIANGULAR FRAME.

If we consider a triangle with two sides which have identical distributed parameters and lengths, and take A and B for the identical sides with subscript 1, we obtain the following frequency equation for an isosceles triangular frame:

$$[\theta_1 + M_{13}(\phi_3 + \theta_3)][\phi_1^2 - \theta_1^2 + M_{13}\theta_1(\phi_3 - \theta_3)] = 0. \qquad (33)$$

As in the rectangular frame each of the factors of Eq. (33) represents a specific type of vibration corresponding to an additional restraint.

Restraint (1).

If we require that the slope of side A at $x_a = 0$ be the same as the slope of side B at $x_b = l_1$, we obtain the following frequency equation:

$$[\theta_1 + M_{13}(\phi_3 + \theta_3)] = 0 \qquad (34)$$

which we recognize as the first factor in Eq. (33). Fig. 10 illustrates the end pattern imposed by this restraint. As in the case of the rec-

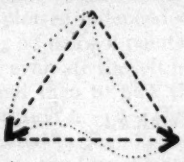


Fig. 10

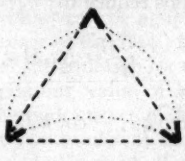


Fig. 11

tangular frame, the heavy dotted lines indicate the undeflected frame, the heavy curved portions indicate the required end pattern, and the light dotted curves illustrate a mode of vibration conforming to this particular end pattern.

The deflection constant values for Restraint (1) are:

$$\left.\begin{aligned}
&\frac{B_2}{A_2} = 0, \qquad \frac{C_2}{A_2} = -N_{13}M_{13}, \\
&\frac{A_1}{A_2} = -\cot k_1 l_1, \qquad \frac{A_3}{A_2} = \coth k_1 l_1, \\
&\frac{B_1}{A_2} = -\csc k_1 l_1, \qquad \frac{B_3}{A_2} = \operatorname{csch} k_1 l_1, \\
&\frac{C_1}{A_2} = N_{13}M_{13}(\csc k_3 l_3 + \cot k_3 l_3), \\
&\frac{C_3}{A_2} = -N_{13}M_{13}(\cot k_3 l_3 + \operatorname{csch} k_3 l_3).
\end{aligned}\right\} \qquad (35)$$

Although, at first glance, there appears to be no relation between the deflection curves of sides A and B, substitution of the deflection constants in the deflection curve equations will reveal that:

$$\left[\frac{X_A}{A_2}\right]_{x_a=x} = -\left[\frac{X_B}{A_2}\right]_{x_b=l_1-x}, \tag{36}$$

so that actually only two sides have to be plotted to determine the form of any mode satisfying Eq. (34).

Restraint (2).

If on the other hand we require that the slope of side A at $x_a = 0$ be the negative of the slope of side B at $x_b = l_1$ we obtain the following frequency equation:

$$[\phi_1^2 - \theta_1^2 + M_{13}\theta_1(\phi_3 - \theta_3)] = 0 \tag{37}$$

which we recognize to be the second factor of Eq. (33). The deflection constant ratios reduce to:

$$\left.\begin{aligned}
&\frac{B_2}{A_2} = \frac{\phi_1}{\theta_1}, \qquad \frac{C_2}{A_2} = N_{13}M_{13}, \\
&\frac{A_1}{A_2} = \frac{\phi_1}{\theta_1}\csc k_1l_1 - \cot k_1l_1, \qquad \frac{A_3}{A_2} = \coth k_1l_1 - \frac{\phi_1}{\theta_1}\operatorname{csch} k_1l_1, \\
&\frac{B_1}{A_2} = \csc k_1l_1 - \frac{\phi_1}{\theta_1}\cot k_1l_1, \qquad \frac{B_3}{A_2} = \frac{\phi_1}{\theta_1}\coth k_1l_1 - \operatorname{csch} k_1l_1, \\
&\frac{C_1}{A_2} = N_{13}M_{13}(\csc k_3l_3 - \cot k_3l_3), \\
&\frac{C_3}{A_2} = N_{13}M_{13}(\coth k_3l_3 - \operatorname{csch} k_3l_3).
\end{aligned}\right\} \tag{38}$$

Substitution of these values into the deflection curve equations reveals that:

$$\left[\frac{X_A}{A_2}\right]_{x_a=x} = \left[\frac{X_B}{A_2}\right]_{x_b=l_1-x}, \tag{39}$$

so that actually detailed plotting is necessary for only two sides of an isosceles triangular frame.

Using the values of the deflection constant ratios in Eq. (38) it can be shown that the slope of side A at $x_a = l_1$ and the slope of side B at $x_b = 0$ are both zero for Restraint (2). This additional point is illustrated in Fig. 11 which shows the end pattern introduced by Restraint (2).

APPLICATION TO A UNIFORM ISOSCELES TRIANGULAR FRAME.

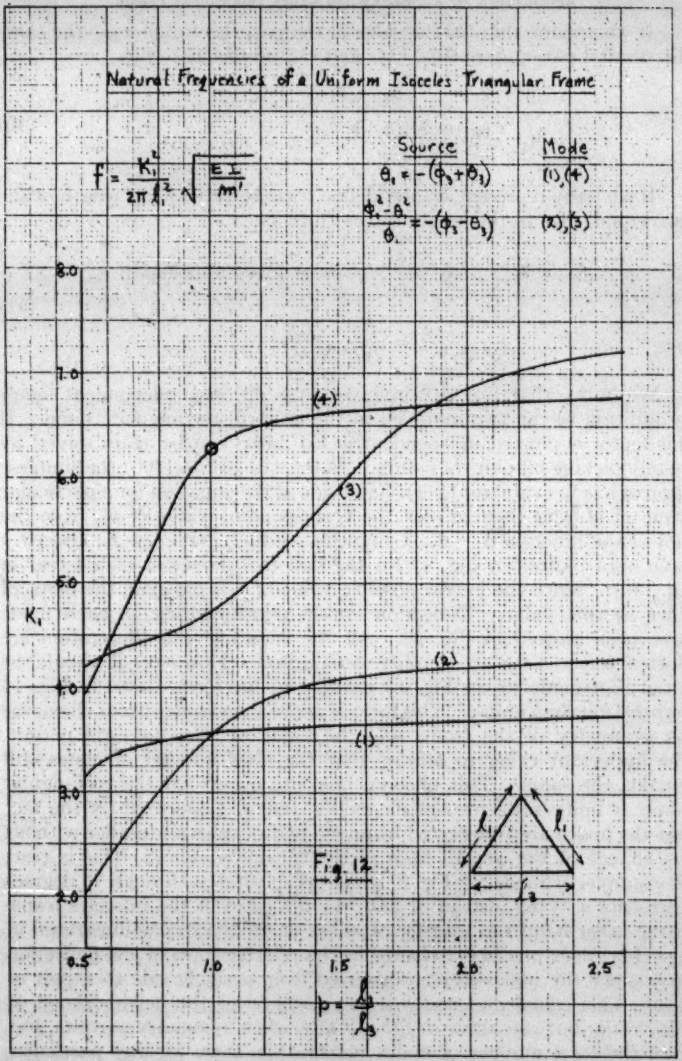
If we assume that all the sides of the isosceles frame have the same distributed parameters Eqs. (34) and (37) reduce to:

$$\left.\begin{aligned} &(a)\quad \theta_1 = -(\phi_3 + \theta_3). \\ &(b)\quad \frac{(\phi_1 - \theta_1)(\phi_1 + \theta_1)}{\theta_1} = -(\phi_3 - \theta_3). \end{aligned}\right\} \tag{40}$$

If we let K represent kl, and define p as the ratio of the length of the two equal sides to the length of the third side:

$$\left.\begin{aligned} &K_1 = kl_1, \qquad p = \frac{l_1}{l_3}, \qquad m = \text{mass/unit length}, \\ &f = \frac{K_1^2}{2\pi l_1^2}\sqrt{\frac{EI}{m}}. \end{aligned}\right\} \tag{41}$$

We notice that Eq. (40) involves an additional independent factor in addition to the two involved in the uniform rectangular frame. If this factor is plotted independently, Eq. (40a) can be again solved by merely picking off pairs of points. In the case of Eq. (40b) the simplest approach is to evaluate the left hand side of the equation for a particular value of kl_1, and then to pick the corresponding value of kl_3 from the plot of the function $\phi - \theta$. Using this method, curves of K_1 against p have been plotted in Fig. 12 for the first two modes of each factor in Eq. (40). Since the lowest possible value of p is one-half, the plots all start at this value. Deflection curves corresponding to each mode have been plotted in Fig. 13. All the modes in this illustration are plotted for $p = 0.8$ to simplify comparison. However, it should be noted that mode (3) in Fig. 13 changes to the form shown in Fig. 14 for p greater than unity. This form of nodal transfer will occur whenever the symmetry of the frame and the mode of vibration will permit. The important thing to note is that the total number of nodes still remains the same. This phenomenon also occurs for the rectangular frame at unity ratio for certain modes, but it is obscured by the fact that the nodes are transferred from one pair of sides to another without any change in the overall form of the deflection curve. This is true, for example, of mode (*d*) Fig. 8, which when rotated through 90 degrees represents $n = 0.5$ instead of 2.0. In the cases of the isosceles frame on the other hand the transfer of nodes at unity ratio as illustrated in Fig. (14a) does produce a change in the relative form of the deflection curve since the nodes were transferred from a single side to a pair of sides. This transfer of nodes is not confined to the neighborhood of unity ratio, but can occur at other ratios when symmetry permits, but again the total number of nodes remains the same for any particular mode. For example, referring to mode (*f*) Fig. 8, a pair of nodes can

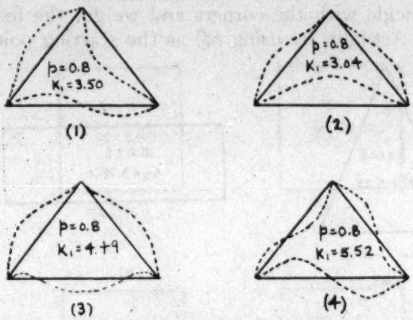

Natural Frequencies of a Uniform Isosceles Triangular Frame
$f = \frac{K_1^2}{2\pi l_1^2}\sqrt{\frac{EI}{m'}}$
Source
$\theta_1 = -(\phi_2 + \theta_2)$
$\frac{\phi_2^2 - \theta^2}{\theta_1} = -(\phi_3 - \theta_3)$
Mode
(1), (4)
(2), (3)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
K_1
(4)
(3)
(2)
(1)
0.5
1.0
1.5
2.0
2.5
$p = \frac{l_2}{l_3}$
l_1
l_1
l_2
Fig. 12.

be transferred from each of the short sides to each of the long sides by increasing n. Fig. (14b) illustrates mode (f) Fig. 7 for $n = 3.0$.

UNIFORM EQUILATERAL FRAMES.

The relations already derived for the more general case of unequal sides do not in general apply to the special case of unity ratio. When we consider equilateral frames we find that some modes disappear,

Natural Modes of an Isoceles Triangular Frame



Source	Mode
$\theta_1 = -(\phi_3 + \theta_3)$	(1), (4)
$\frac{\phi_1^2 - \theta_1^2}{\theta_1} = -(\phi_3 - \theta_3)$	(2), (3)

Fig. 13

others approach limiting forms and the rest remain fundamentally unchanged.

Square Frame.

For a square frame Eq. (28) reduces to:

$$\left.\begin{array}{ll} \text{(a)} & \phi + \theta = 0, \\ \text{(b)} & \phi - \theta = 0, \\ \text{(c)} & \theta = 0, \end{array}\right\} \qquad (42)$$

where Eqs. (28c) and (28d) both reduce to (42c).

If we let n approach unity in Fig. 7, K_2 appears to approach π for mode (a) but actually there is no value in the neighborhood of π which satisfies the corresponding Eq. (42b). The same condition exists for mode (e) Figs. 7 and 8, except that in this case K_2 appears to approach 2π as n approaches unity. Again there is no solution of Eq. (42a) in the vicinity of 2π. This means that, for the symmetrical type of frame support assumed, modes (a) and (e) cannot appear for a perfectly square uniform frame. For this reason values of K_2 for $n = 1$ on the curves for modes (a) and (e) in Fig. 7 have been encircled.

The remainder of the modes have been plotted in Fig. 15. As n approaches unity the nodal points on the longer sides of mode (d) Fig. 8 approach the corners. In the limiting case of unity ratio the nodal points coincide with the corners and we get the form shown in (d') of Fig. 15. Actually by using (d) as the starting point we would

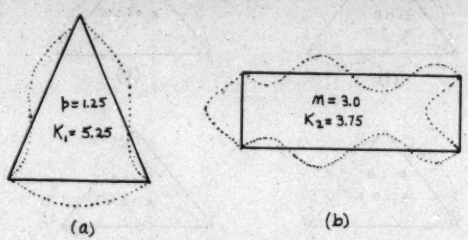


Fig. 14

get an envelope with all the sides deflected inward, but this in itself has no significance as it is the relative form of the deflection curve which is important. Fig. 15 (d') was plotted a half cycle out of phase so to speak to illustrate this point. Another mode which loses a pair of nodal points on each of a pair of sides in the limiting case of unity ratio is mode (h) of Fig. 8. In this case Fig. (15h') has been plotted so that we can obtain it directly from the plot of mode (h) of Fig. 8 by moving the nodal points to zero.

An examination of modes (b) and (c) of Fig. 8 would lead us to suspect that they would coincide for unity ratio. They do, but otherwise they are fundamentally unchanged as illustrated by (b'), (c') of Fig. 15. The same condition exists for modes (f) and (g) of Fig. 8 as illustrated by (f'), (g') of Fig. 15.

Equilateral Triangular Frames.

For the special case of $p = 1$ Eq. (40) reduces to:

$$\left.\begin{array}{ll} \text{(a)} & (\phi + 2\theta) = 0, \\ \text{(b)} & (\phi - \theta)(\phi + 2\theta) = 0, \end{array}\right\} \qquad (43)$$

and contains the following independent relations:

$$\left.\begin{array}{ll}(a) & \phi = -2\theta, \\ (b) & \phi = \theta.\end{array}\right\} \tag{44}$$

If we examine the curve of mode (4) Fig. 12, we see that K_1 apparently approaches the value 2π as p approaches unity. Actually, however, Eq. (44a) has no solution in the immediate neighborhood of 2π. For this reason the value for unity ratio has been encircled. It should be pointed out, however, that the absence of this mode for unity ratio is predicated upon a perfectly equilateral, uniform and symmetrically

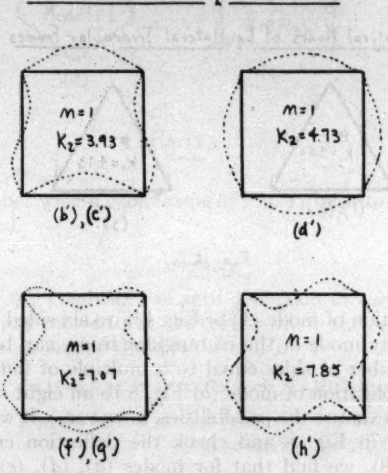


Fig. 15

supported frame, and it is quite likely that it will still appear for frames that deviate even slightly from these conditions.

If we consider mode (3) Fig. 13 and its counterpart in Fig. 14 for p greater than unity we could probably guess the limiting form of the mode for unity ratio since both figures must approach an identical form. If we start with p less than unity using mode (3) Fig. 13 the two nodes on the third side approach the corners as p approaches unity. On the other hand if we started with p greater than unity the individual nodes on each of the equal sides approach the corners as p approaches unity. In either case the limit for unity ratio has the form (3′) in Fig. 16.

For unity ratio modes (1) and (2) of Fig. 13 have the same form as

illustrated by (1′), (2′) of Fig. 16, which is not much different from mode (2) but differs from mode (0) by one nodal point.

GENERAL APPLICATION OF RESULTS.

Although in the preceding developments we apparently have determined the natural frequencies for frames of two particular shapes, i.e., the rectangle and the triangle, the results are actually applicable to an infinite number of frames.

The most obvious application of the results for the triangular frame would be to frames with a number of sides equal to a multiple of three. For this application one-third of the sides would have the length l_3 and the remaining two-thirds would have the length l_1. Fig. (17a) illus-

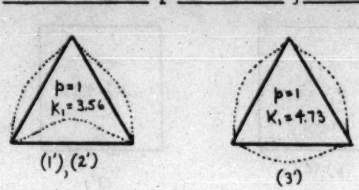


Fig. 16

trates the application of mode (1) of Fig. 13a to six sided frames. In a similar manner any mode of the rectangular frame can be applied to a frame with a number of sides equal to a multiple of four. Fig. (17c) illustrates the application of mode (*b*) Fig. 8 to an eight sided frame.

This does not exhaust the possibilities, however. If we examine the modes illustrated in Fig. 8 and check the deflection curve relations Eqs. (15) and (18), we find that for modes (*a*), (*d*), (*e*), and (*h*) the deflections of opposite sides of the frame are identical. This means that the results obtained for these modes can be applied to any frame having an even number of sides. Fig. (17b) illustrates the application of mode (*a*) Fig. 8 to a six sided frame.

If we examine the equilateral forms we find that mode (*d*′) Fig. 15 and mode (3′) Fig. 16 represent a type of vibration in which all sides have identical deflection curves. This mode of vibration is, therefore, applicable to any uniform equilateral frame. Fig. (17d) illustrates the application of this mode to a pentagonal frame.

Finally since we have zero slope at the adjoining ends of the two equal sides for modes (2) and (3) of Fig. 13 we can apply these modes to a three sided frame with built-in ends replacing the corner. Fig. (17e) illustrates the application of mode (2) Fig. 13 to this type of frame.

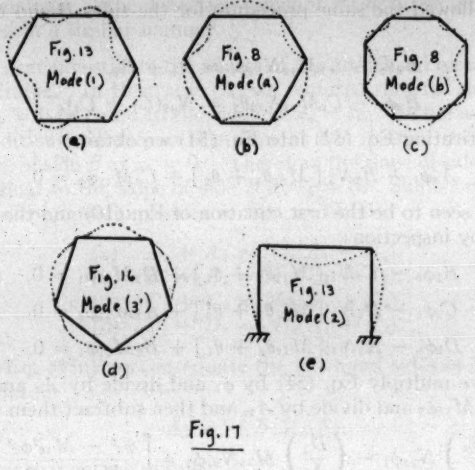


Fig. 17

The figure number within each frame of Fig. 17 indicates the mode upon which it is based.

APPENDIX.

If we apply the conditions of zero deflection at each end and equal slopes and bending moments at the corner where they adjoin, we obtain the following relations for the sides A and B:

$$A_1 \sin k_1 l_1 + A_2 (\cos k_1 l_1 - \cosh k_1 l_1) + A_3 \sinh k_1 l_1 = 0, \quad (45)$$

$$A_1 \cos k_1 l_1 - A_2 (\sin k_1 l_1 + \sinh k_1 l_1) + A_3 \cosh k_1 l_1 = N_{21}(B_1 + B_3), \quad (46)$$

$$- A_1 \sin k_1 l_1 - A_2 (\cos k_1 l_1 + \cosh k_1 l_1) + A_3 \sinh k_1 l_1 = - 2M_{21}N_{21}B_2. \quad (47)$$

Adding and subtracting Eq. (45) and Eq. (47), and solving for A_1 and A_3 we get:

$$A_1 = B_2 M_{21} N_{21} \csc k_1 l_1 - A_2 \cot k_1 l_1, \quad (48)$$

$$A_3 = A_2 \coth k_1 l_1 - B_2 M_{21} N_{21} \operatorname{csch} k_1 l_1. \quad (49)$$

Adding Eq. (48) and Eq. (49):

$$A_2 \theta_1 - B_2 M_{21} N_{21} \phi_1 = A_1 + A_3. \quad (50)$$

Substituting Eq. (48) and Eq. (49) into Eq. (46) and simplifying:

$$A_2 \phi_1 - B_2 M_{21} N_{21} \theta_1 = N_{21}(B_1 + B_3). \quad (51)$$

If we followed the same procedure for the sides B and C we would obtain:

$$B_2\theta_2 - C_2M_{12}N_{12}\phi_2 = B_1 + B_3, \tag{52}$$

$$B_2\phi_2 - C_2M_{12}N_{12}\theta_2 = N_{12}(C_1 + C_3). \tag{53}$$

By substituting Eq. (52) into Eq. (51) we obtain:

$$A_2\phi_1 - B_2N_{21}[M_{21}\theta_1 + \theta_2] + C_2M_{12}\phi_2 = 0. \tag{54}$$

This can be seen to be the first equation of Eq. (10) and the others can be written by inspection:

$$B_2\phi_2 - C_2N_{12}[M_{12}\theta_2 + \theta_1] + D_2M_{21}\phi_1 = 0, \tag{55}$$

$$C_2\phi_1 - D_2N_{21}[M_{21}\theta_1 + \theta_2] + A_2M_{12}\phi_2 = 0, \tag{56}$$

$$D_2\phi_2 - A_2N_{12}[M_{12}\theta_2 + \theta_1] + B_2M_{21}\phi_1 = 0. \tag{57}$$

If now we multiply Eq. (54) by ϕ_1 and divide by A_2 and multiply Eq. (56) by $M_{12}\phi_2$ and divide by A_2, and then subtract them we obtain:

$$\left(\frac{B_2}{A_2}\right)N_{21}\phi_1 - \left(\frac{D_2}{A_2}\right)M_{12}N_{21}\phi_2 = \left[\frac{\phi_1^2 - M_{12}^2\phi_2^2}{M_{21}\theta_1 + \theta_2}\right]. \tag{58}$$

Multiplying Eq. (57) by $M_{12}N_{21}$ and dividing by A_2 we get:

$$\left(\frac{D_2}{A_2}\right)M_{12}N_{21}\phi_2 - M_{12}(M_{12}\theta_2 + \theta_1) + \left(\frac{B_2}{A_2}\right)N_{21}\phi_1 = 0. \tag{59}$$

Adding Eq. (58) and Eq. (59) and re-arranging:

$$\frac{B_2}{A_2} = M_{12}N_{12}\left[\frac{(\theta_1 + M_{12}\theta_2)^2 + \phi_1^2 - M_{12}^2\phi_2^2}{2\phi_1(\theta_1 + M_{12}\theta_2)}\right]. \tag{60}$$

This can be seen to be the first deflection constant ratio given in Eq. (12). By substituting Eq. (60) in Eq. (54) and Eq. (57) and simplifying we obtain:

$$\frac{C_2}{A_2} = \left[\frac{(\theta_1 + M_{12}\theta_2)^2 - (\phi_1^2 + M_{12}^2\phi_2^2)}{2M_{12}\phi_1\phi_2}\right], \tag{61}$$

$$\frac{D_2}{A_2} = N_{12}\left[\frac{(\theta_1 + M_{12}\theta_2)^2 - (\phi_1^2 - M_{12}^2\phi_2^2)}{2\phi_2(\theta_1 + M_{12}\theta_2)}\right]. \tag{62}$$

By dividing Eq. (48) and Eq. (49) by A_2 and using the reciprocal relation between $M_{21}N_{21}$ and $M_{12}N_{12}$ we obtain:

$$\frac{A_1}{A_2} = \left(\frac{\csc\ k_1l_1}{M_{12}N_{12}}\right)\left(\frac{B_2}{A_2}\right) - \cot\ k_1l_1, \tag{63}$$

$$\frac{A_3}{A_2} = \coth\ k_1l_1 - \left(\frac{\csc\ k_1l_1}{M_{12}N_{12}}\right)\left(\frac{B_2}{A_2}\right). \tag{64}$$

The remainder of the deflection constant ratios given in Eq. (12) can be obtained in a similar manner.

Let us next investigate the effects of the additional restraints placed upon the frame. In Restraint (A) we required that for the side A the slope at $x_a = 0$ be equal to the slope at $x_a = l_1$. At the same time the frame conditions still require the slope of side A at $x_a = l_1$ to be equal to the slope of side B at $x_b = 0$. Therefore the slope of side A at $x_a = 0$ must be equal to the slope of side B at $x_b = 0$. Similar reasoning can be used for the remaining sides so that we obtain:

$$\left.\begin{array}{ll} \text{(a)} & A_1 + A_3 = N_{21}(B_1 + B_3), \\ \text{(b)} & B_1 + B_3 = N_{12}(C_1 + C_3), \\ \text{(c)} & C_1 + C_3 = N_{21}(D_1 + D_3), \\ \text{(d)} & D_1 + D_3 = N_{12}(A_1 + A_3). \end{array}\right\} \tag{65}$$

Using Eq. (65a) we can equate the left hand sides of Eqs. (50) and (51) and obtain:

$$A_2 = -B_2 M_{21} N_{21}. \tag{66}$$

In a similar manner we obtain:

$$B_2 = -C_2 M_{12} N_{12}, \tag{67}$$

$$C_2 = -D_2 M_{21} N_{21}, \tag{68}$$

$$D_2 = -A_2 M_{12} N_{12}. \tag{69}$$

But Eq. (66) is the condition for the bending moment of side A at $x_a = 0$ to be the negative of the bending moment of side B at $x_b = 0$. Since the frame requires the bending moment of side A at $x_a = l_1$ to be equal to the bending moment of side B at $x_b = 0$, we have the bending moment at the two ends of side A equal to the negative of each other. The same is true of the other sides of the frame for Restraint (A).

If we use Eqs. (66) and (67) in conjunction with Eq. (54) we obtain:

$$[(\phi_1 + \theta_1) + M_{12}(\phi_2 + \theta_2)] = 0, \tag{70}$$

which is identical with Eq. (13). It should be realized that we can obtain the same relation by using Eqs. (52), (53), or (54) with the appropriate pair of Eqs. (66), (67), (68), and (69).

Restraint (B) follows the same pattern except that the bending moments are identical at the two ends of each side. Due to the restraints already present in the frame, therefore, if we require the slopes at the two ends of a side to be the same we automatically make the bending moments the negative of each other, while if we require the slopes to be the negative of each other, we automatically make the bending moments the same at the two ends.

If we consider Restraint (C) the relationships between the slopes give us:

$$\left.\begin{array}{ll} \text{(a)} & A_1 + A_3 = -N_{21}(B_1 + B_3), \\ \text{(b)} & B_1 + B_3 = N_{12}(C_1 + C_3), \\ \text{(c)} & C_1 + C_3 = -N_{21}(D + D_3), \\ \text{(d)} & D_1 + D_3 = N_{12}(A_1 + A_3), \end{array}\right\} \tag{71}$$

which lead to the following bending moment relations:

$$\left.\begin{array}{ll} \text{(a)} & A_2 = B_2 M_{21} N_{21}, \\ \text{(b)} & B_2 = -C_2 M_{12} N_{12}, \\ \text{(c)} & C_2 = D_2 M_1 N_{21}, \\ \text{(d)} & D_2 = -A_2 M_{12} N_{12}. \end{array}\right\} \tag{72}$$

Using Eq. (72a) and Eq. (72b) together with Eq. (54) we obtain:

$$[\phi_1 - \theta_1 - M_{12}(\phi_2 + \theta_2)] = 0, \tag{73}$$

which is the same as Eq. (19). Restraint (D) can be evaluated in a similar manner.

The frequency equations and deflection constants for a triangular frame are treated in much the same manner as for the rectangular frame.

It may be of interest to verify Eq. (36). Using the constants in Eq. (35) we get:

$$\left[\frac{X_A}{A_2}\right]_{x_a=x} = -\cot k_1 l_1 \sin k_1 x + \cos k_1 x - \cosh k_1 x + \coth k_1 l_1 \sinh k_1 x. \tag{74}$$

On the other hand:

$$\left[\frac{X_B}{A_2}\right]_{x_b=l_1-x} = -\csc k_1 l_1 \sin k_1 (l_1 - x) + \operatorname{csch} k_1 l_1 \sinh k_1 (l_1 - x), \tag{75}$$

which when expanded simplifies to:

$$\left[\frac{X_B}{A_2}\right]_{x_b=l_1-x} = \cot k_1 l_1 \sin k_1 x - \cos k_1 x + \cosh k_1 x - \coth k_1 l_1 \sinh k_1 x, \tag{76}$$

which is the negative of Eq. (74). The relation expressed by Eq. (39) can be verified in a similar manner.

We will next prove that the slopes at the corner where the equal sides adjoin for the modes of Eq. (37) are actually zero. The slope of side B at $x_b = 0$ for the isosceles frame:

$$\left[\frac{dX_B}{dx_b}\right]_{x_b=0} = k_1(B_1 + B_3). \tag{77}$$

Using the values in Eq. (38) we get:

$$\left[\frac{dX_B}{dx_b}\right]_{x_b=0} = k_1 A_2 \left[\csc k_1 l_1 - \operatorname{csch} k_1 l_1 + \frac{\phi_1}{\theta_1} (\coth k_1 l_1 - \cot k_1 l_1) \right]. \quad (78)$$

$$\left[\frac{dX_B}{dx_b}\right]_{x_b=0} = k_1 A_2 [\csc k_1 l_1 - \operatorname{csch} k_1 l_1 + \operatorname{csch} k_1 l_1 - \csc k_1 l_1] = 0. \quad (79)$$

Since the frame conditions require that the slope of side A at $x_a = l_1$ be equal to the slope of side B at $x_b = 0$ we have:

$$\left[\frac{dX_A}{dx_a}\right]_{x_a=l_1} = \left[\frac{dX_B}{dx_b}\right]_{x_b=0} = 0, \quad (80)$$

as stated in the discussion of this type of vibration.

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Research on Guided Missiles.—Rockets that will surpass the German V-2 in speed, distance, accuracy, and effectiveness are the primary objectives of a program of guided missile research being conducted jointly by General Electric and the Army Ordnance Department, according to an announcement by R. S. Neblett, Manager of G-E's Federal and Marine Divisions.

"The research program was started in November, 1944," he said. "G-E scientists were sent to Europe to study captured V-2 rockets, prior to V-E Day."

"Special facilities have been provided by the Army at the Company's Schenectady Works for testing new types of rockets and motors," Mr. Neblett stated. "Meanwhile, Company scientists and officials have been assisting with test firings of the V-2 rockets at the Army Ordnance Proving Grounds at White Sands, N. M."

Mr. Neblett disclosed that power propulsion research is along the lines of rocket engines, ram-jets, and a combination of both. The ram-jet is simply a cylinder which compresses the air through speed in flight, adds and ignites fuel in the combustion chamber, and derives forward motion from the thrust produced by expanding gases which blast through the nozzle. Dependent on oxygen from the air, ram-jets are subject to range-altitude limitation; their ceiling cannot much exceed the maximum altitude for aircraft.

Development and use of new, more powerful fuels creates metallurgical problems, according to G-E scientists. These materials must have greater strength, withstand the corrosive actions of these fuels, and resist melting caused by generation of tremendous friction heat in flight. Rocket fuel, as used by the Germans, is liquid oxygen and alcohol, mixed in the combustion chamber.

Among flight requirements for the new rocket missiles will be the problem of trajectory. On long, flat trajectory, close to the earth's surface, the increased force of gravity affects flight. In such circumstances, additional powers must be provided without reducing the explosive charge in the "war head." High-angle trajectory, propelling the missile far above the atmosphere, may result in disintegration on the return-to-ground.

Now, for determining the nature of the upper air, special instruments replace the "war head." Weighing as much as 1,800 pounds, these instruments, through radio impulses, relay to the ground temperatures, pressures, cosmic rays, spectrograph studies, and other dynamic effects on the missile. By forcible ejection from the body of the missile, delicate recording instruments, bearing invaluable data, can be parachuted to the ground undamaged. Previously, such information was limited to the ceiling altitude of the free balloon, 21 miles. Now, the rocket propelled missile pierces the ionosphere over 100 miles above the earth.

R. H. O.

NOISE AND OSCILLATIONS IN HOT CATHODE ARCS.*

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ABSTRACT.

Gas discharges generate oscillations superimposed on random noise of several megacycles bandwidth. Positive ions oscillate in two distinct regions of the discharge, i.e., the plasma and the potential minimum at the cathode. The disturbances produced by the ion motions appear as voltage variations between the electrodes. Oscillations generated in the plasma are usually below 400 kc./sec., while "cathode" oscillations are about 700 kc./sec. As the discharge current is increased, the amplitude of the plasma oscillation increases while its frequency decreases. The random noise also depends on current, increasing rapidly as the Townsend discharge changes into an arc. After arc conditions are fully established, the noise changes slightly with current. Probe studies show that the noise voltage between probe and cathode increases rapidly as space charge conditions change to permit acceleration of electrons to the probe. When the probe collects positive ions, the noise as viewed on an oscilloscope is coarse, i.e., lacking in high frequency components. An explanation of the noise generation may be found in the random fluctuations of space charge at the electrodes. Calculations based on this theory give voltage fluctuations of the observed order of magnitude.

INTRODUCTION.

This paper covers an investigation of the voltage fluctuations that occur in gas discharges. The research was largely confined to phenomena occurring in hot-cathode arcs in the pressure range from 10^{-2} to 2 mm. Hg. These voltage fluctuations are of two kinds, usually coexistent, viz., non-sinusoidal oscillations and random variations (electrical noise). A brief summary of the state of knowledge at the time this investigation was started will be instructive.

Appleton and West [1] studied ionic oscillations in striated glow discharges. These oscillations were found with both hot and cold cathodes. They found the frequency proportional to pressure and to the anode voltage. The pressure was in the range 5 to 250 microns. The currents were of the order of 3 ma. The observed frequencies were in the range 10^3 to 10^5 c.p.s.

Compton and Eckart [2,3] studied oscillations in hot-cathode arcs in helium and in mercury vapor in the pressure range 3 to 12 mm. Hg.

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[1] E. V. Appleton, and A. G. D. West, *Phil. Mag.*, **45**, 879 (1923).

[2] C. Eckart, and K. T. Compton, *Phys. Rev.*, **24**, 97 (1924).

[3] K. T. Compton, and C. Eckart, *Phys. Rev.*, **25**, 139 (1925).

The amplitude of these voltage variations was of the order of the ionization potential, and it was believed that they were brief surges rather than true oscillations. The frequency was observed to decrease as the current was decreased or as the filament temperature was increased. Oscillations which appear to have been of the relaxation type were studied by Newman,[4] Clay [5] and Gill.[6]

Thomson [7] encountered oscillations that appear to have been relaxations and in addition found oscillations that appeared to be generated within the tube and which were independent of the circuit constants. He also observed ionic oscillations that appeared to be generated by the relaxation disturbance.

Tonks and Langmuir [8] observed oscillations in ionized gases from 0.5 to as high as 1,000 mc./sec. The high-frequency oscillations were interpreted as oscillations of the electrons in the plasma. It was shown mathematically that these electron oscillations should have a frequency in cycles per second given by

$$f_e = \left(\frac{ne^2}{\pi m_e}\right)^{\frac{1}{2}} = 8980n^{\frac{1}{2}}, \qquad (1)$$

where n is the electron concentration, e is the electronic charge and m_e is the mass of the electron. The low-frequency oscillations were interpreted as positive-ion oscillations in the plasma. These positive-ion oscillations for singly-charged ions were shown to have an upper frequency limit in *c.p.s.* given by

$$f_p = \left(\frac{ne^2}{\pi m_p}\right)^{\frac{1}{2}}, \qquad (2)$$

where n is the positive-ion concentration (approximately equal to the electron concentration in the plasma), e is the electronic charge and m_p is the mass of the ion. They found that for mercury ions the upper frequency limit of the positive ion oscillations is of the order of 1.5 mc./sec.

Pardue and Webb[9] found the frequency of the oscillations (20 to 150 kc./sec.) in a hot-cathode "glow discharge" to be independent of the external circuit constants. For their tube the frequency decreased as the gas pressure was increased from 0.03 to 0.09 mm. Hg, increased with the anode voltage, and increased with the filament current. These

[4] F. H. Newman, *Phil. Mag.*, **47**, 839 (1924).
[5] R. E. Clay, *Phil. Mag.*, **50**, 985 (1925).
[6] E. W. B. Gill, *Phil. Mag.*, **8**, 955 (1929).
[7] J. J. Thomson, *Phil. Mag.*, **11**, 697 (1931).
[8] L. Tonks and I. Langmuir, *Phys. Rev.*, **33**, 195 (1929); I. Langmuir, *Proc. Nat. Acad. Sci.*, **14**, 627 (1928).
[9] L. A. Pardue and J. S. Webb, *Phys. Rev.*, **32**, 946 (1928).

oscillations were thought to be plasma-ion oscillations as analyzed by Tonks and Langmuir.[8]

Much lower frequencies, below 1.5 kc./sec. were investigated by Kniepkamp [10] for gas pressures from 0.5 to 5 mm. Hg. In this case the oscillations were regular and pulse like and the arc drop was found to vary in magnitude between the ionization potential and the voltage of the "low-voltage" arc. These were not relaxation oscillations, but were characteristic of the discharge. Kniepkamp suggested that the variations may be due to regular changes of the discharge from one conducting path to another. The outward diffusion of ions would decrease the space-charge maximum of the "low-voltage" arc [11] along one path and thus cause the voltage across the discharge to rise and a new path to be established. This view was also held by Funk and Seeliger [12] who studied hot-cathode discharges at pressures from 0.1 to 4 mm. Hg. In their work the frequencies observed extended to 25 kc./sec.

There appears to have been no systematic study made of the noise developed in gas discharges. Langmuir and Tonks [8] observed electrical noise in connection with their investigation of plasma oscillations. Occasionally in the radio literature it is mentioned that gas tubes are "noisy," which may mean random noise or objectionable oscillations, and a hot-cathode discharge tube has been used as a source of noise in the production of artificial speech.[13,14] Ballantine [15] investigated the noise developed in vacuum tubes that had some residual gas (pressures from 10^{-5} to 6×10^{-4} mm. Hg), and showed that the presence of small amounts of residual gas increased the noise developed in the tube above the normal shot noise for the same current. He found that the noise output increased as the three-halves power of the plate voltage and directly as the ratio of gas pressure to gas temperature. This author states that true plasma discharge conditions did not exist, i.e., the phenomenon was that associated with the ionization process occurring in the Townsend type of discharge.[16] The ionization-noise current was of relatively low level, of the order of ten times the shot-noise current for a gas-free diode. The spectrum of the ionization noise showed a gradual decrease above 1 mc./sec. Oscillations were observed between 650 and 700 kc./sec. These oscillations were believed to be caused by positive ions oscillating in the potential minimum at the cathode.[17]

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[10] H. Kniepkamp, *Zeits. f. tech. Phys.*, **17**, 397 (1936).
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[14] H. Dudley, R. R. Riesz and S. S. A. Watkins, J. FRANKLIN INST., **227**, 739 (1939).
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[16] J. D. Cobine, "Gaseous Conductors," pp. 143–159, McGraw-Hill Book Co. (1941).
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EXPERIMENTAL METHOD.

The noise and oscillations were studied in the present research by means of heterodyne-type spectrum analyzers,[18,19,20] and wideband cathode-ray oscillographs. The three analyzers covered the frequency range from 25 c.p.s. to 9 mc./sec. and were carefully designed to avoid clipping the noise, since clipping would result in a false spectrum.[21] Bandwidths of 60 c.p.s., 200 c.p.s., 11 kc./sec., and 33 kc./sec. were available among the analyzers. The cathode-ray oscillographs had bandwidths of 2.5 and 10 mc./sec. In order to have the measuring device present a high impedance to the gas tube, a buffer amplifier (cathode follower) was always used to connect the analyzer to the gas tube.

The noise voltage indicated by an r.m.s. device varies as the square root of the bandwidth of the measuring device if the spectrum is flat.[22,23] The bandwidths of the analyzers were narrow enough so that the spectrum being measured could be considered flat over this small frequency interval. Since each analyzer had a different effective bandwidth, it was necessary to establish a common base to correlate the data. After a preliminary survey to determine the approximate range of voltages to be measured, it was decided to take the zero-db. level as 10 μv./(kc./sec.)$^{\frac{1}{2}}$. The spectrum curves give the r.m.s. noise voltage in a 1-kc. bandwidth centered at any frequency. When both noise and oscillations are present, as is the case with gas tubes operated without external magnetic field, the oscillation amplitude may be as much as 40 db. (100 times) greater than the noise level. Under these conditions in order to prevent the oscillations from overloading the analyzer and distorting the entire spectrum, the input signal was attenuated to a safe value, and the amplification of the frequency band actually being measured was increased. It was not found practical to measure noise with an accuracy greater than ± 0.5 db.

EXPERIMENTAL RESULTS.

The spectrum of the output voltage of the low-pressure arc with externally heated cathode varies widely among tubes of different types. Figure 1 gives spectra typical of these discharges. Each curve shows the continuous spectrum representing the electrical noise together with numerous strong peaks of the oscillations. Evidently the spectrum of a given tube is markedly affected by the magnitude of the discharge current. In general it may be stated that the noise level of frequencies less

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[22] F. E. Terman, "Radio Engineers' Handbook," p. 477, McGraw-Hill Book Co. (1943).

[23] E. B. Moullin, "Spontaneous Fluctuations of Voltage," Oxford, p. 17 (1938).

than the first peak is relatively flat. For Curve *A* the noise level is 80 db. above the shot-noise level for a diode with a 3000-ohm plate resistance. After the second peak, the noise level decreases rapidly with frequency. The amplitude of the high-frequency components of the noise decreases according to the relation $V(f) = V_0 f^{-\alpha}$ where α varies from 2.3 to 3.8 for different types of tubes. This is an actual decrease in the amplitude of the high-frequency components of the generated voltage and not an effect of external shunting capacitance across the tube. If the load impedance is kept at 5,000 ohms or greater, the tube acts as a constant-potential generator of noise voltage and the true spectrum of this voltage may be obtained.

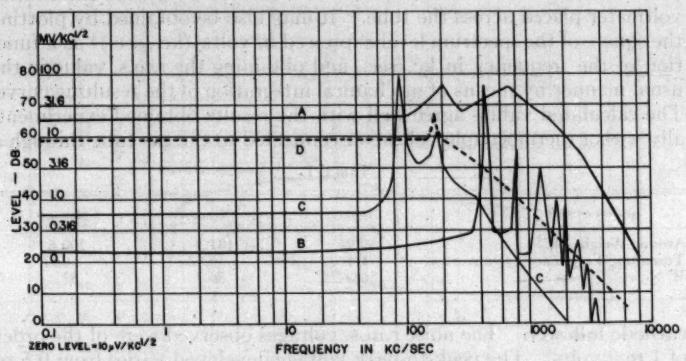

FIG. 1. Typical spectra of gas tubes.

A— 884 Tube, anode current = 40 ma. (argon)
B— 884 Tube, anode current = 3 ma.
C—2050 Tube, anode current = 3 ma. (xenon)
D— 816 Tube, anode current = 125 ma. (mercury)

Thiede [24] has studied theoretically and experimentally the spectrum obtained from various kinds of random pulses, such as obtained from an alpha-particle source and microphone noise. He obtained spectra for which the low-frequency level was constant, and the high-frequency level decreased at a rate dependent on the shape of the pulse. The region of transition from low to high frequency characteristics was determined by the time constant of the pulse decay. In fact, the transition frequency was equal to the reciprocal of the time constant. The spectra shown in Fig. 1 are similar to those obtained by Thiede if one disregards the superimposed oscillations. It will be noted that for each gas represented there is a characteristic transition frequency.

[24] H. Thiede, *E. N. T.*, 13, 84 (1936).

Table I gives the values of the atomic weight, transition frequency, and their product for each gas. It is clear that the transition frequency varies inversely as the atomic weight.

When the oscillations are very steady, the peaks may be quite accurately defined by the analyzer. These conditions hold for Curve *B* of Fig. 1, where the fundamental and its harmonics are quite definite. Under other conditions, as in Curve *A* of Fig. 1 there is a random shift in frequency of the fundamental, as indicated by the much broader peak. For some types of tube, variations in noise amplitude of as much as ± 10 db. were observed, whereas for other types the variation was as low as ± 1 db. from tube to tube. If the oscillations are not present, the r.m.s. noise voltage can be measured by a wide band r.m.s. voltmeter placed across the tube. It may also be obtained by plotting the square of the spectrum level expressed in volts/(kc./sec.)$^{\frac{1}{2}}$ as a function of the frequency in kc./sec., and obtaining the r.m.s. value in the usual manner by means of mechanical integration of the resulting curve. The calculated values agree well with the results obtained experimentally with a thermocouple voltmeter connected to the gas tube through a

TABLE I.

Gas	Argon	Xenon	Mercury
Atomic Weight (W)	39.94	131.3	200.6
Transition Frequency (f)-mc.	1.4	0.43	0.26
$W \times f$	56	56.3	52

cathode follower. The noise r.m.s. voltages observed were of the order of 1 to 2 volts. The peak-to-peak voltage developed varies from 0.5 to 0.9 of the arc drop.

A summary of the tube drop, noise and oscillation characteristics for the 884 tube is presented in Fig. 2 as functions of anode current. The tube drop curve (e_a) exhibits two regions characteristic of the hot-cathode discharge. Below 1.1 ma. a "Townsend" discharge amplifies the initial thermionic current from the cathode. This discharge does not have a plasma, although for currents greater than about 0.2 ma. a glow is present at the anode. At 1.1 ma. the discharge changes to the arc with the formation of a plasma throughout most of the discharge path. Curve *A* is a line drawn through the battery voltage, E_{bb}, with a slope equal to the negative of the load resistance to indicate the path of transition from "Townsend" discharge to arc. Since the load was a resistance and was kept constant as the arc current was increased by increasing the supply voltage, the locus of all arc-voltage variations will be along a line having this slope and drawn through the mean current position wherever located. The frequency (f) and amplitude of the fundamental component of the oscillations are also shown in the figure. The noise is represented by the level at 50 kc./sec. The level at this

frequency is characteristic of the entire noise spectrum below the lowest oscillation component. The peak output voltage ($\hat{V}$) as measured by a General Radio vacuum-tube voltmeter, type 726A, represents the contributions of both noise and oscillations. It is important to remember that, although these data may be considered typical for the low-pressure arc, considerable numerical variations are to be expected even among tubes of the same type.

At low currents the fundamental oscillation is at frequencies higher than 500 kc./sec. This oscillation was detected for currents as low as

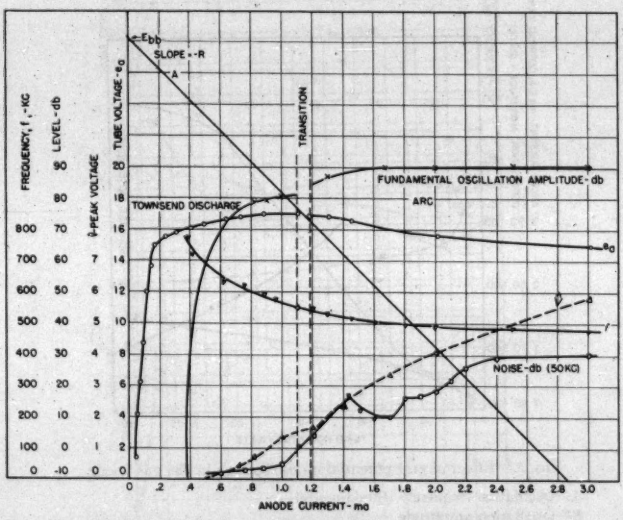

FIG. 2. Characteristics of 884 gas triode (argon filled).

0.38 ma., which is well down in the "Townsend" discharge region. Hence this cannot be a plasma oscillation,[8] since a plasma or arc column does not exist for this type of discharge. This would appear to be an oscillation of the type observed by Ballantine [15] and interpreted as the oscillation of positive ions in the potential minimum at the cathode. This cathode oscillation is probably present over the entire current range studied. Continuous observation over the entire range is made difficult by the appearance at about 5 ma., of an oscillation of much lower frequency with many strong harmonics. Figure 1, Curve *A*, shows the cathode oscillation at 750 kc./sec. together with the lower-frequency peak at 200 kc./sec. Experiments with a magnetic field described

elsewhere [25] indicate that the lower frequency oscillation is a plasma-ion oscillation.

The amplitude and frequency of the plasma oscillation are affected by negative voltages on the grid as shown in Fig. 3 for the 884 type of gas tube. The grid potential is the voltage measured between grid and cathode. The oscillation frequency increases continuously as the grid is made more negative up to the point of extinction of the discharge (see Curve *A*). The amplitude of the oscillation varies irregularly with

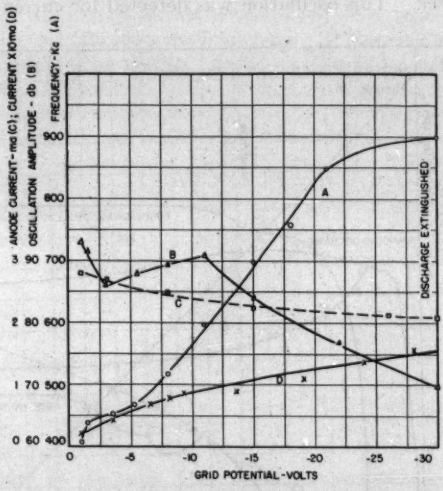

FIG. 3. Effect of grid potential on oscillations in 884 gas triode.

A—Oscillation frequency (fundamental).
B—Oscillation amplitude.
C—Anode current (constant supply voltage and fixed load resistance).
D—Positive ion current to grid.

a general trend towards lower values as the bias becomes more negative. The decrease in anode current (Curve *C*) is due to an increase in arc voltage. The increase in arc drop is due partly to the constriction of the arc column by the positive ion space charge around the grid and partly to the loss of positive ions to the grid, a loss which must be made up by increased ionization. The increased ionization is probably the cause of the increased frequency of the plasma-ion oscillation frequency (Eq. 2). The limiting frequencies given in Fig. 3, Curve *A* (400 kc./sec.

[25] C. J. Gallagher and J. D. Cobine, *Phys. Rev.*, **70**, 113 (1946); *J. Applied Physics*, **18**, Jan. (1947).

and 900 kc./sec.), are in plausible agreement with the plasma oscillation equation. This increase in frequency requires that the ion density increase by a factor of about 5.

The noise output is very low in the "Townsend" region (Fig. 2) and is probably the shot noise associated with the emission current amplified

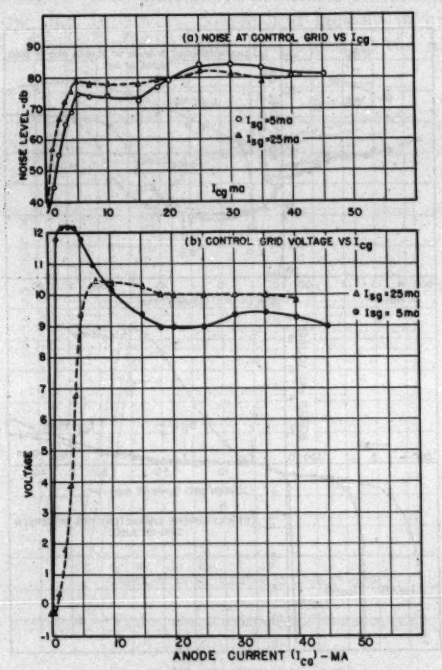

FIG. 4. Control grid characteristics of 2050 gas tetrode (control grid as anode).

(*a*) Noise measured between control grid and cathode (noise measured at 5 kc./sec.).
(*b*) Control grid voltage vs. control grid current.

I_{cg} = control grid current.
I_{sg} = screen grid current.

by ionization.[15] In the transition region there is a marked increase in noise as the arc plasma is established. The noise level is of the same order for tubes differing in plasma volume by a factor greater than 2,000, i.e., for miniature tubes and large arc tubes, such as the 816, 866, and experimental tubes of large volume and a discharge path 3 inches long.

Considerable information concerning the relation between noise and the discharge parameter was obtained by studying the 2050 screen-grid thyratron. In this study it was found convenient to use the control-grid as anode, with the screen grid as a "probe" surrounding the discharge. The spectrum (Fig. 1) is similar to that of the 884 but the oscillations are of lower frequency (50–100 kc./sec.). The control-grid

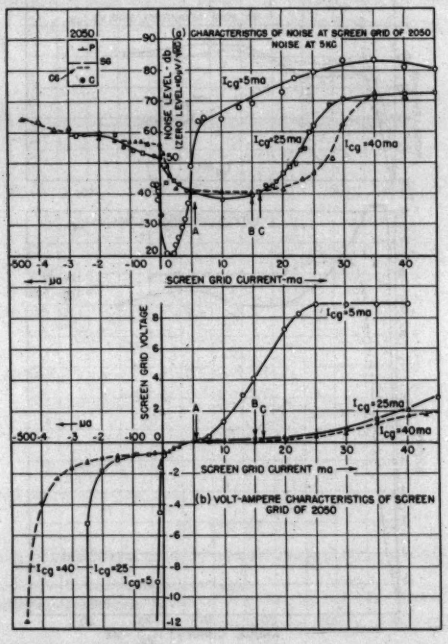

FIG. 5. Screen grid characteristics of the 2050 gas tetrode.

(*a*) Characteristics of noise measured between screen grid and cathode (noise measured at 5 kc./sec.).

(*b*) Volt-ampere characteristics of screen grid. I_{cg} = control grid current.

(anode) to cathode noise and voltage are presented in Figs. 4a and 4b as functions of control-grid current for two values of screen-grid current. The noise was measured at 5 kc./sec., since the spectrum is flat below the lowest oscillation and thus the level of the 5 kc./sec. noise is characteristic of the entire low-frequency region. As in the 884, the noise is relatively independent of arc current after arc conditions are fully established.

With the main discharge established between cathode and control grid, the noise between cathode and screen grid is highly dependent on screen-grid current (Fig. 5a). Examination of the volt-ampere characteristics of the "probe" (Figs 5b and 6) show that for very negative voltages, where only positive-ions are being collected, the noise level is fairly high. Examination of this noise on an oscilloscope and analyzers show that it is very coarse and lacking in high frequency components. As the probe is made less negative, the noise decreases

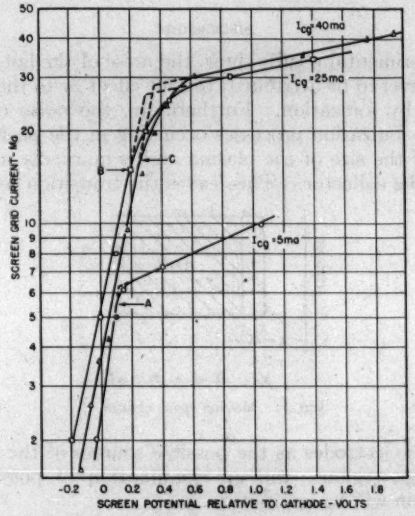

FIG. 6. "Probe" characteristics of 2050 screen grid.

A—Control grid current = 5 ma.
B—Control grid current = 25 ma.
C—Control grid current = 40 ma.

and levels off, remaining relatively constant. The level in this region depends on the main arc current. In this range of probe potential, the probe is collecting increasingly large electron currents, although the electrons are still moving in against a retarding potential. At the points marked *A*, *B*, and *C* on Figs. 5b and 6, the electrons start to be accelerated. In the vicinity of these points the noise is exhibiting a rapid increase. In the accelerating region, double layers are formed at probe surface, which consist of an electron concentration directly adjacent to the surface, and a positive-ion concentration at the plasma

edge.[26] Thus, the maximum noise level is obtained when the double layer is established. It will be noted from Fig. 4 that changing the screen current has relatively little effect on the noise or voltage of the arc itself. The amplitude of the noise at the screen grid is substantially the same as that at the control grid if the current densities are the same. For example, in Fig. 4a, the noise at the control grid for $I_{cg} = 5$ ma. and $I_{sg} = 25$ ma. is 78 db. In Fig. 5a, the noise level at the screen is 78 db. at $I_{sg} = 25$ ma. and $I_{cg} = 5$ ma. These currents have about the same ratio as the collecting areas, so that the current densities are the same.

DISCUSSION

As the experimental results show, the noise of the hot-cathode arc is of too high a level to be attributed to shot effect or to the amplification of shot effect by ionization. Furthermore, the noise cannot be due directly to the ionization processes occurring in the plasma, since it is independent of the size of the plasma and is markedly affected by the conditions at the collector. This leaves the transition regions between

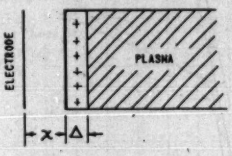

FIG. 7. Moving space charge.

plasma and the electrodes as the possible sources of the noise. These are space-charge regions, and an examination of possible processes occurring therein will be instructive.

As an example of noise generation by space-charge fluctuations, consider the idealized charge layer in Fig. 7. Let σ be the charge per square centimeter of area facing the electrode. Then if there are no other charges present, the electrode will have a surface-charge density of $(-\sigma)$. The plasma will be assumed a perfect conductor which bounds the charge layer on one side. If the positive-ion charge density n_p in the layer of thickness Δ be assumed to be the same as in the plasma, then

$$\sigma = n_p e \Delta,$$

where e is the electronic charge. The voltage difference between the electrode and the charge layer considered as plane-parallel plates is then

$$V = \frac{\sigma}{C} = 4\pi n_p \Delta e x, \tag{3}$$

[26] I. Langmuir, *Phys. Rev.*, **33**, 969 (1929).

where x is the distance between the electrode and the space-charge layer and C is the capacitance per unit area. If x be assumed to have the form

$$x = d + af(t), \tag{4}$$

where "d" is the average value and "a" is the amplitude of the variational motion, then the amplitude of the fluctuating voltage may be calculated. The following values may be taken as reasonable:

$n_p = 10^{10}$ ions/cm.^{3}, $d = 0.1$ cm., $a = 0.01$ cm., $\Delta = 0.01$ cm.

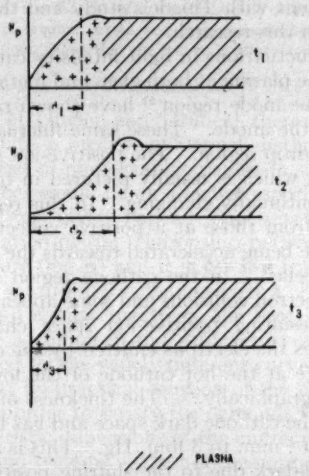

FIG. 8. Possible space charge distributions.
t_1, t_2, t_3, represent different times.

Then

$$V = 18 + 1.8f(t) \qquad \text{(volts)}. \tag{5}$$

This, then, would be the voltage developed between the electrode and the plasma if the entire space-charge facing the electrode fluctuates as a unit. If each small part of the space-charge varied independently, the net variational voltage would be zero. The r.m.s. voltages observed were of the order of 1 to 2 volts with peak-to-peak voltages of about 85 per cent. of the arc drop. The simple mechanism suggested appears to be capable of developing voltages of the magnitude of those observed. Of course the picture of the phenomena given in Fig. 7 is too idealized, since the space charge is not sharply defined, but is a function of distance from the electrode. Figure 8 shows three possible successive

space charge distributions with effective distances d_1, d_2, d_3, which could produce somewhat the same effect as the idealized picture.

Any such configuration can be produced almost instantaneously by ionization processes. The disappearance will be at a slower rate as determined by the mobility of the ions. The mobility of the ions varies inversely with their atomic weight. From Thiede's analysis, the rate of decay of these individual ionization pulses determines the frequency at which the spectrum level starts to decrease. It has already been shown (Table I) that for gas discharge spectra, the transition frequency varies inversely as the atomic weight so that the above picture of noise generation is consistent with Thiede's study and the characteristics of the noise observed in this research.

Low-frequency fluctuations in light intensity can be observed visually at the edge of the plasma at both glass and metal surfaces. Photographic studies of the anode region [27] have shown random fluctuations in the ionization at the anode. These same fluctuating space charges exist in the cathode-drop region. The positive-ion space charge in the cathode-drop region, which is usually pictured in terms of its average state, is probably continually changing. In this region conditions are somewhat different from those at a positive collector. At a positive collector electrons are being accelerated towards the electrode while the positive ions are repelled. In the cathode region some positive ions are formed by the electron collisions and some appear by diffusion from the plasma. The resulting positive-ion space charge produces the field which accelerates the electrons emitted by the cathode.

A semi-dark space at the hot cathode of the low-pressure arc has been observed photographically.[28] The thickness of this dark space is about twenty times the cathode dark space and has been observed over the pressure range 10^{-2} mm. to 3 mm. Hg. This is a scattering region and is probably semi-dark due to the shifting position of the cathode edge of the luminous plasma and consequent variations in the thickness of the ionization zone. Thus conditions are quite complicated at the cathode and depart widely from the simple analysis usually given. Since the entire discharge depends on the cathode-drop region for its maintenance, these complicated fluctuations must be taken into account in any exact analysis.

[27] T. Takamine, T. Suga and A. Yanagihara, *Scientific Papers Inst. of Phys. and Chem. Research (Japan)*, **21**, 26 (1933).

[28] M. J. Druyvesteyn and N. Warmholtz, *Physica*, **4**, 51 (1937.)

AN OBJECTIVE METHOD OF SPECIFICATION FOR SENSITOMETRIC CURVES OF PHOTOGRAPHIC PAPERS.

BY

C. A. MORRISON.

Communication No. 1110 from the Kodak Research Laboratories.

ABSTRACT.

A system of numerical specification of the sensitometric curves of photographic papers is described. Comparison of original curves and their reproductions, made by a new curve duplicator from specifications, shows that the system is satisfactory for production control.

INTRODUCTION.

Since the time of Hurter and Driffield there have been repeated attempts to formulate a precise, numerical method of expressing the sensitometric qualities of photographic papers in such a manner that a single term or number would be an adequate specification of the sensitometric curve of any given paper. The sensitometric curve is a graphical representation of the relationship between reflection density (D_R) and log exposure (log E) and it is *per se* a complete and sufficient specification of the sensitometric characteristics. If these curves were graphs of simple and similar functions, the functional equation alone would therefore be a complete and satisfactory specification but the functions represented by the widely different curves are always so complex and varied that no practical solution of the problem has been attained; therefore, in order to satisfy the demands for numerical evaluation of the characteristic dimensions of the curve, resort has been made to various devices and systems. Most of these proposals are not unique in all respects so that one and only one curve can satisfy the specification because the very important factors which govern the shape of the curve are insufficiently expressed.

Investigation (1) of this problem has provided a system which satisfies the requirements of specification, but goes beyond the needs of a system for production control because of the inclusion of subjective factors. Modification of this system by elimination of the subjective factors gives a simple and easily applied method of specification of the sensitometric curve which is discussed and described in the following pages.

Examination of a typical sensitometric curve (Fig. 1) of photographic papers shows that it may be considered to consist of three parts: (1) a portion slowly rising from the lowest density called the toe, (2) a central region of inflection, which passes to (3) in which the gradient returns to zero. The relative extent and relative shape of each part

may and do vary greatly among the papers currently manufactured so that, although the ends of two curves may have the same respective positions on the coordinate scale, the curve may be very different (thick *vs.* thin curve, Fig. 1). These differences are usually quite significant and must be accounted for in a system of specification.

Most systems heretofore proposed utilize as characteristic values the obvious differences between selected points near the extremities of the curve on the respective coordinates, viz., the density scale and the log exposure scale. The systems have differed usually by the choice of

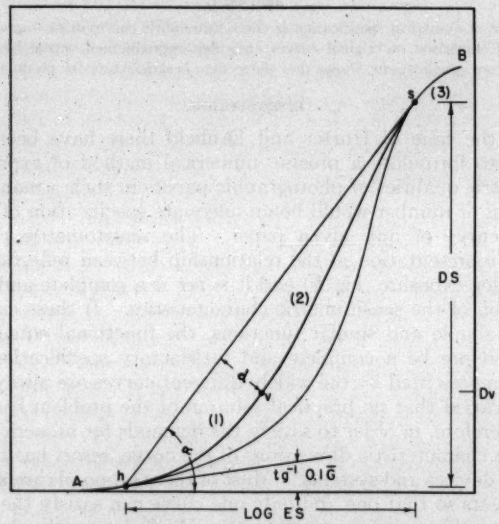

FIG. 1. Geometry for specification of the sensitometric curve.

the criteria of the limits, which have been defined in such terms as fixed density values, fixed or fractional gradients, just-noticeable differences of density from maximum and/or minimum density, etc. The establishment of these limits does not, however, in any way determine the course which the curve follows from one limit to the other. The specifications are therefore incomplete and insufficient.

Many proposals of systems of specification have or imply an inherent subjective factor, a measure of "contrast." Contrast has been expressed in terms of one or more of the various characteristic values derived from the curve for the purpose of specifying the curve on a scale of subjective values, but these systems likewise do not provide explicit

specifications of the shape of the curve. Although the inclusion of a measure of the little-understood factor of contrast may be a desirable addition to specifications from the standpoint of the important subjective aspects of tone reproduction, it is not at all necessary for the practical application of specifications to manufacturing control.

Manufacturing control of the quality and grading of photographic papers is generally based directly or indirectly on the visual judgment of prints made from "standard negatives." When the sensitometric curve of any accepted standard paper is obtained, it represents the standard curve for that paper. Any other paper which has the same sensitometric curve and is similar in all other physical aspects will be identical, and will produce under similar conditions of exposure and processing identical prints that are visually indistinguishable from the accepted standard print. It is only necessary therefore that correspondence of the curve itself with the standard curve be required to establish the similarity of sensitometric qualities of any other paper with the selected standard. Such a procedure does not involve necessarily any measurement of subjective factors but only a physical similarity or identity of curves. Any system, which will provide for the numerical evaluation of certain and sufficient dimensions of the sensitometric curve and on the basis of the derived numbers will permit the precise reconstruction of one and only one curve uniquely similar to the original from which the numerical specifications were obtained, becomes a satisfactory system of specification. Such a system will now be described.

METHOD OF SPECIFICATION.

Consider a typical sensitometric curve (A, v, B, Fig. 1) of a contemporary paper which has, for example, a useful portion, hs, as determined by the proposal of Jones and Nelson(2), in which the criteria of the points h and s are, respectively, such that h is that point at which the gradient of the curve is equal to 0.1 $\bar{G}$ and s is that point at which the gradient is equal to $\bar{G}$, where $\bar{G}$ is the average gradient between the two limiting points. A third point, v, is that point on the curve between h and s at which the gradient is also $\bar{G}$. There are now three points on the curve for which the values of their coordinates may easily be derived from the D_R-log E coordinate system and their respective gradient values may be obtained by construction or by computation from the coordinate differences. The best smooth curve drawn through three such points with the gradients according to specification is a very close approximation to the original curve from which the values were derived.

A complete set of specifications for a curve may now be written in the following terms:

$$\text{Log } E_s, D_s,$$
$$\text{Log } E_v, D_v,$$
$$\text{Log } E_h, D_h.$$

Derived values provide the quantitative terms which give the numerical evaluation of the sensitometric characteristics:

Log exposure scale	Log ES	Log E_s − Log E_h
Density scale	DS	$D_s - D_h$
Average gradient	$\bar{G}$	$tg\alpha = DS/\text{Log } ES$
Speed	S	$10^4/E_s$
Vertex ratio	V	$100\,\frac{D_v - D_h}{DS}$
Arcuance	A	d (Fig. 1) measured from the line hs to the point v normal to hs, expressed in coordinate scale units times 100.

The first four terms are common to sensitometric practice. Vertex ratio indicates a quality of the shape of the curve in terms of the relative position of the vertex, v, along the density scale. From the geometrical relationship, which exists between the characteristic curve of the negative and that of the positive material when perfect tone reproduction is attained, it will be found that the complementary curves of typical negatives are those in which the vertex ratio is very much higher than that found in the average paper curve. Therefore, in the case of two papers similar in other respects except the vertex ratio, the one having a vertex ratio nearer to that of the negative will be the more desirable.

Arcuance, a measure of the degree of bowing, has been defined (1) in subjective terms as the ratio of the contrast (Ω) of the curve to the contrast (ω) of the straight line, hs, and in this form it provides a measure of the subjective effect which results from the bowing of the curve. With the elimination of the subjective factors, further study of the correlation of the various terms with the original evaluation of arcuance shows that the use of the sagittal distance, d, alone as expressed in the previous tabulation is sufficient for practical use, particularly when papers of the same density scale are compared. Of two ordinary curves similar except in arcuance, the one having an arcuance more nearly that of the negative will be in general practice the more satisfactory paper.

A CURVE DUPLICATOR.

The practicability of this method as a means of specification has been demonstrated by the use of the device, a curve duplicator, shown in Fig. 2. It consists of an arm, A, rotatable about an axis, which is at the origin of the coordinate scales on the base plate. This arm may be locked in position by turning the knurled nut. The arm, which corresponds to the line, hs, of Fig. 1, carries two sliding members with vertical rods, V and S, adjustable to the positions, v and s. The bearing of the arm, A, carries a rod, H, whose axis occupies the position, h. Each of these rods, V, S, and H, has a slot in the upper end to receive the norm-

ally flat strip of spring steel, 0.005 inch thick and 0.5 inch wide. The dimensions of the spring are not critical. The slots in the rods for the v and s positions are always parallel to the length of the arm. The slot in the axle, H, is always at an angle with the abscissa axis such that its tangent is 0.1 times the tangent ($\bar{G}$) of the angle which the arm forms with the base line. This relationship is maintained for any position of the arm by means of the pantograph linkage connected with the lower extension of the arm and it may be adjusted for different ratios by

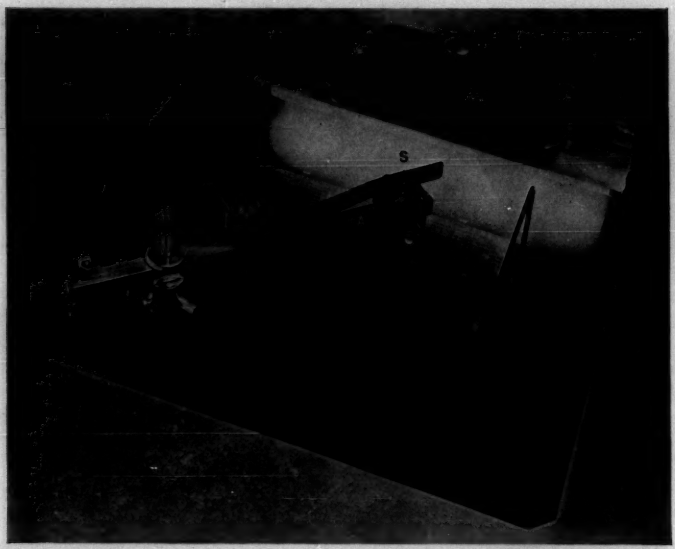

FIG. 2. Curve duplicator.

changing the position of a pivot on the pantograph. The slot, v, is carried on a rod through the slider and is adjustable along a line normal to the length of the arm. The axes of the rods, v and s, as projected to the coordinate grid are located by pointers on the undersides of the sliders.

When the pointers are adjusted to the DS-log ES coordinate scales on the base according to the derived values, the three vertical rods occupy positions which satisfy the primary specification values and each slot automatically has its prescribed slope. The shape of the strip of steel between h and s becomes that of the curve from which the specifica-

tions were derived. A notch in the steel strip indicates the position of the point, *h*, and two wires, each on opposite sides of the strip, mark the position of the point, *s*.

A clamp to hold a sheet of coordinate paper is mounted on a double slide on the back of the instrument to provide adjustments to the proper coordinate positions. The positions of the double slide may be fixed by suitable clamping screws. If a sheet of sensitometric graph paper, with its axes respectively parallel to those of the coordinate system of

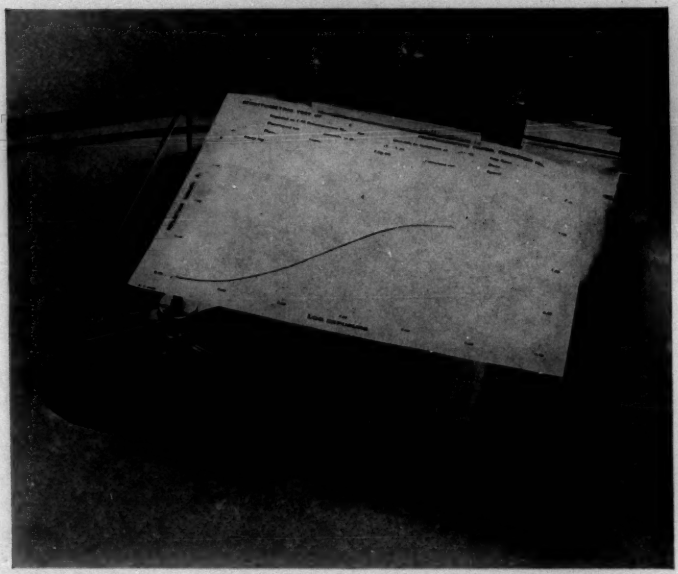

FIG. 3. Curve duplicator set for comparing extant curve.

the device, is laid over the spring with the notch in the steel strip placed at the correct position for *h*, a copy of the curve correctly placed on the paper may be drawn by placing a piece of carbon paper, face down, over the paper and rubbing a straight-edge over it. The notch and the wires at *h* and *s* mark respectively the limits of the curve by causing breaks in the line. A fluorescent lamp recessed under the sliding paper clamp illuminates the base plate, and it also forms a shadow of the steel strip in contact with the underside of the paper to permit correct positioning and also direct comparison with a curve already drawn (Fig. 3).

ACCURACY OF DUPLICATION OF CURVES.

The accuracy of the instrument in duplicating curves from the specifications has been tested with curves of 116 standard papers, which cover the entire range of manufactured papers. It was set for each paper according to the specifications and the respective original curve was placed over the spring. The maximum deviation of the spring from the original curve was measured in terms of scale units. Most of the deviations occurred in the upper part of the inflected region and were always positive. Five per cent. of the deviations occurred in the toe and were very small. The size and frequency of the differences are as follows:

Deviation in Scale Units	Frequencies %
0.00	60
0.01	22
0.02	10
0.03	5
0.04	3

The differences in the first three categories (92 per cent. of the curves) are of the same order and frequency as those which may exist between curves made from strips taken from various samples of the same record. If the spring is set for a mean position with respect to the most divergent curves, no difference is greater than 0.02. This amount of difference is usually not noticeable in a print.

This method fulfills the requirements for a system of specification of the sensitometric characteristics of photographic papers and is amply precise for use as a measure of control of present production.

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ROCHESTER 4, N. Y.,
August 19, 1946.

Mammoth Truck Dynamometer.—A mammoth 60-ton, 1,000 hp., 45-m.p.h. combination truck-dynamometer, built through the cooperative efforts of the General Electric Company, the Knuckey Truck Company of San Francisco, and the U. S. Army Ordnance Department, will soon appear on the Aberdeen Proving Ground, Md., it has been revealed.

Primary purpose of the 40-foot, gasoline-electric giant dynamometer is to test the tractive effort of large Army wheeled and tracked combat vehicles over various types of terrain. As such, it rates a maximum drawbar pull of 60,000 pounds at speed as low as two miles per hour.

Employed as a prime mover to tow test equipment, such as heavy field pieces and trailers, to ascertain tractive rolling resistance, the truck rates continuously 38,000 pounds tractive effort at a speed of 2.5 m.p.h. As a truck functioning merely to propel itself, the vehicle is capable of 60,000 pound tractive effort and a top speed of 45 m.p.h.

The 1,000-hp. drive is provided by two 500-hp. engine-generator power plants, each consisting of a Ford tank engine and a General Electric traction generator. Each generator supplies power to two traction motors which drive the wheels through sprocket and chain drive.

The dynamometer can be operated with one or both power plants. Engine speed is controlled by an electro-hydraulic governing system. Automatic overload and current limit electrical circuits protect the engines and generators from damage. Amplidyne exciters furnish excitation for motors and generators.

The cab of the massive vehicle accommodates a driver and load-control operator. When operating as a truck the driver controls the vehicle with two foot pedals, one for power and one for braking. Being an electric-drive vehicle, there is no gear shifting. Reverse of direction is controlled by a manually operated switch on the instrument panel.

Dynamometer or load-absorbing operation is controlled by both the load-control operator and the driver. The driver steers the vehicle and operates the air brakes when required. The dynamometer or load-control operator controls the electrical load on the towed vehicle. Mounted on a small panel, directly in front of the operator, are a control switch, which makes the electrical load circuits operative, and three control knobs. The load-control operator can increase or decrease dynamometer load absorption by means of the three knobs.

Operational performance of the dynamometer is recorded by drawbar stain gage equipment and elaborate speed, time, and fuel consumption instruments.

R. H. O.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

CYCLE-TESTING LABORATORY FOR AIRCRAFT BATTERIES.

A new laboratory for battery research has been constructed at the National Bureau of Standards with the cooperation of the Bureau of Aeronautics, Navy Department. Equipped with completely automatic controls for cycling batteries, this laboratory offers a unique method of testing batteries under conditions simulating those encountered in practice—conditions that are particularly important for aircraft batteries. Such problems as maximum electrical capacity per unit weight, charging techniques, and the effect of these techniques on battery life, will be investigated.

In present day aircraft, the storage battery plays an important role in the operation of electrical equipment such as radio, radar, and lighting. It also serves as a source of standby power in the operation of the gun turrets, bomb bays, and retractable landing gear. Since every pound added to a plane increases the takeoff distance, weight is also of major concern. Hence, obtaining maximum capacity per unit weight of the battery with reasonable life expectancy is the principal aim of the experimental tests conducted at the Bureau of Standards.

The equipment in the laboratory provides for discharging the batteries through constant resistance and recharging them either at constant potential or at constant current, as desired. The cycling may be performed automatically for batteries under conditions of room temperature, extreme heat, sub-zero temperatures, and "flight" vibration.

Six similar cycling units have been installed, each operated independently by a master electric clock. Each unit consists essentially of a control cabinet, two motor-generator sets, and the necessary resistors. From one to six 24-volt batteries may be cycled on each unit, the only requirement being that the batteries on a given unit have approximately equal rated capacities. When charging at constant potential, the batteries are connected in parallel; when charging at constant current or when discharging, they are connected in series. These connections are made automatically.

An important feature of the laboratory is the control cabinet. One is provided for each cycling unit and contains a program machine for automatic cycling, sequence relays, contactors (aircraft type), and a copper oxide rectifier which supplies 24-volt current for operating the switching devices. Three single-phase transformers located in alter-

* Communicated by the Director.

nate control cabinets supply 115-volt alternating current for the rectifiers. The transformers are supplied by the respective phases of the alternating current supply for the motor-generator sets.

Included on the control cabinets are follower clocks mounted on the back of the program machines, the various voltmeters, ammeters, cycle-counters, and ampere-hour meters, and the trunk line terminals. The trunk lines provide for transferring one or more batteries from the normal test position to a refrigerator, an electric furnace, or a vibrating machine to approximate conditions encountered in practice. The program machine of each unit is mounted on the back of a hinged panel which swings outward to make it accessible. The sequence relays, contactors and shunts are located inside the cabinets with the sequence relays in two horizontal rows near the top, the contactors in the middle, and the shunts in two rows near the bottom of the wall.

The motor-generators have squirrel-cage induction type motors and compound-wound generators equipped with voltage regulators and reverse current relays. The machines are two-bearing units with both rotors and fan mounted on the same shaft. The motors are rated at 11 horsepower and the generators at 30 volts, 250 amperes maximum. Two machines operate in parallel for each cycling unit when charging at constant potential. Each motor-generator set is provided with an independent line starter so that either machine may be disconnected when the peak load is passed. When charging at constant current the batteries are switched to the 115/230-volt direct current power supply.

When discharging the batteries, natural circulation by flue action is depended upon for removing the heat from the resistor housing. The resistor units are made of nichrome ribbon for high rates of discharge, while helical coils of nichrome wire provide for intermediate and low rates of discharge and for charging at constant current. The resistor units are mounted on sheet metal panels and tapped for one to six batteries. The desired current values can be obtained approximately from combinations of the resistor units in series and parallel. Fine adjustments of one ampere or less may be obtained from rheostats mounted in the control cabinets. The maximum safe discharge rate for each cycling unit is about 250 amperes.

The cycle-testing control begins with the master clock which provides electrical impulses at intervals of one minute for advancing all of the program machines. A 24-volt battery maintained on trickle charge provides energy for winding the clock and for actuating the program machines.

Each program machine carries three endless tapes which pass under contact fingers and control six program circuits. The tapes are printed with the time of day and night and make one revolution in twenty-four hours. A program is set up by punching holes near the edges of the tapes in accordance with the time schedule for the required operations.

The six control circuits in each program machine control the following operations, respectively:

Circuit 1. (*a*) Starts motor-generator sets and connects batteries in parallel on constant potential charge. (*b*) Stops motor-generator sets and disconnects batteries.

Circuit 2. (*a*) Connects batteries in series on constant current charge. (*b*) Disconnects batteries.

Circuit 3. (*a*) Connects batteries in series on high-rate discharge. (*b*) Disconnects batteries.

Circuit 4. (*a*) Connects batteries in series on intermediate rate or low rate of discharge. (*b*) Disconnects batteries.

Circuit 5. (*a*) Short-circuits a portion of the charging resistor for constant current charging at normal rate. (*b*) Opens short-circuit on resistor for finishing rate.

Circuit 6. (*a*) Operates cycle counter. (*b*) Disconnects one motor-generator set after peak load is passed when charging at constant potential.

The control circuits in any program machine can be used independently or concurrently in any order depending upon the program which has been set up. Toggle switches in the machines provide for switching to manual operation when desired, and skip pins which lift the contact fingers provide for omitting the program for specified intervals of time. Interlocking relays prevent the connection of batteries in series and parallel at the same time in the event of an error in manual switching or a failure of a sequence relay. Pilot lights on the control panel indicate when batteries are being charged at constant current. A "no-voltage" disconnect with manual reset prevents overloads which might be caused by the simultaneous starting of the motor-generator sets after an interruption in the alternating current power.

Extensive experimental tests are now under way in the cycle-testing laboratory. It is believed that final and conclusive data from these tests will give an accurate comparison between the two methods—the constant potential and constant current—of charging batteries and their effect on the life of batteries.

The general plan of the laboratory was developed by C. L. Snyder, National Bureau of Standards, with the cooperation of Lt. Kenneth Dawkins of the Bureau of Aeronautics. Engineers of the Standard Electric Time Company worked out the details for the automatic controls and constructed the control cabinets. The resistor units were designed and constructed by members of the National Bureau of Standards, who also designed the layout for the laboratory and installed the equipment.

VACUUM TUBE ACCELERATION PICKUP.

A new electronic tube, in appearance much like an ordinary radio tube, but capable of measuring accurately the rapidly changing accelerations to which various parts of an airplane are subjected in flight, has been developed in the Bureau's Engineering Mechanics Laboratory, under the direction of Dr. Walter Ramberg. The new tube, known as the vacuum tube acceleration pickup, is also proving useful in such applications as measurement of accelerations in portions of the body of "dummy" pilots and living subjects when subjected to critical acceleration during crash landings or seat-ejections from jet-propelled airplanes.

The vacuum tube acceleration pickup takes advantage of the effect of acceleration on the relative position of the electrodes in the tube. This is in contrast to the design of conventional tubes where this effect is suppressed as much as possible because it leads to electrical noise or microphonics. The tube contains a fixed, indirectly heated cathode with two plates, one on either side. The plates are elastically mounted to deflect in response to acceleration normal to the plane of the plates. Deflection of the plates causes a change in plate current proportional to the acceleration and such changes in current are recorded on a standard oscillograph.

The elastic mounting of the plates was designed to give a fundamental frequency of about 800 cycles per second, in order to make the tube capable of recording accelerations with frequencies up to 200 cycles per second. Tests conducted at the Bureau on the latest type tubes (of which 95 have been built) have proved them to have the desired natural frequency with an output proportional to acceleration normal to the plates. The output for an acceleration of the order of 10 times gravity was of sufficient magnitude to record directly on a standard oscillograph without requiring an auxiliary amplifier.

The fundamental frequency of the plate structure for bending normal to the plate was computed by treating the structure as an elastic system with a concentrated mass at the center of gravity of the plates and distributed masses in the supporting wires, which are subject to flexure and torsion by the vibration of the plate normal to its plane. The diameters and lengths of the wires were computed to give a fundamental frequency in this mode in the range of 750 cycles per second. The natural frequencies of vibration in other modes were estimated to be much higher.

The fundamental frequency was determined experimentally by connecting the pickup into a Wheatstone bridge with a cathode ray oscilloscope, tapping the pickup with a pencil or finger, and matching the output on the oscilloscope by the output of known frequency for an audio-oscillator.

The high output and high frequency of the vacuum tube make it possible to record rapidly varying accelerations without an amplifier, a

desirable feature in flight tests of small airplanes where space and weight are at a premium. It also simplified tests in the laboratory when amplifying channels are not readily available. The high capacity of the pickup is advantageous in recording accelerations over a wide range and in preventing damage from sudden high accelerations. The linearity of the pickup permits the filtering out of high frequency response, regardless of amplitude, and produces a record proportional to acceleration.

The development of the vacuum tube acceleration pickup was begun in 1941 for the Navy's Bureau of Aeronautics and was carried on with the cooperation of Sylvania Electric Products, Inc. In addition to experiments for the Army and Navy, the tubes are being tested under various service conditions by several flight laboratories to determine the full scope of their usefulness. In the meantime, they have already proved successful at the Bureau for measuring accelerations of a model airplane during drop tests.

FOREIGN SCIENTIFIC AND ENGINEERING RELATIONS.

With the end of the war relations with scientific laboratories in other countries were renewed. During the summer of 1946, twelve members of the Bureau staff visited other nations; eighteen Bureau engineers were (and are now) located at various stations in the Pacific; and another engineer was recalled by the Army to aid in the organization of a Standards Bureau in Korea.

In the six months from May 1 to November 1, 176 representatives of 35 countries visited the Bureau. Included among these visitors were 10 directors of research institutions whose programs are as broad as that of this Bureau and 9 directors of specialized research institutions, as well as research engineers, university professors, government officials, industrialists, and guest laboratory workers.

International conferences and scientific meetings were resumed. Dr. E. C. Crittenden, Associate Director, has just returned from Paris where he attended a meeting of the International Committee on Weights and Measures, the first since the war. Various matters relating to electrical, photometric, and other standards were considered and plans were made for the first post-war meeting of the International Conference on Weights and Measures.

Dr. H. L. Dryden, Associate Director, recently visited the leading European aeronautical laboratories and attended both the International Technical Congress and the International Congress for Applied Mechanics. Dr. Dryden was elected one of four representatives from the United States on the international committee of the latter. In England Dr. Dryden also conferred with the Director of Building Research and

the Director of the National Physical Laboratory on problems of mutual interest.

Dr. Walter Ramberg, chief of the Engineering Mechanics Section, visited the Structures laboratories at the Royal Aircraft Establishment, National Physical Laboratory, and General Electric Laboratory in England; the laboratories of the Technical University of Delft, the National Luchtvaartlaboratorium in Amsterdam, and the Philips Natuurkundig Laboratorium in Eindhoven; the laboratory of Professor Homes in Mons, Belgium; the Laboratorie des Travaux Publiques in Paris; the E. Material-pruefüngs and Verschsanstalt, Zürich; and the laboratories of A. W. Huggenberger in Zürich.

W. D. Appel, chief of the Bureau's Textiles Section and head of the Textiles Unit, Technical Industrial Intelligence Division, Office of Scientific Services, spent eight weeks in Germany and four weeks visiting laboratories concerned with textiles and high polymer chemistry in Switzerland, Sweden, The Netherlands, and England. He obtained German scientific and technical war-time publications needed by the Bureau Library and brought back first-hand knowledge of textile testing methods and research abroad. The heads or representatives of textile research institutions jointly supported by industry and government in Sweden, Switzerland, The Netherlands, and England have visited the Bureau since his return.

D. R. Miller, Bruce L. Wilson, and L. B. Tuckerman, spent several days in Ottawa, Canada, discussing with representatives of the National Research Council of Canada, the Canadian Standards Association, and United States manufacturers, details of the United States' participation in a program of research on screw threads. An outgrowth of the efforts toward unification of screw-threads on the three Nations, this research is a cooperative effort between the National Physical Laboratory in England, the National Research Council of Canada, and the National Bureau of Standards.

Dr. William Blum, chief of the Chemistry Division's Electrochemistry (Electroplating) Section, attended a conference of electrochemists at Toronto in October, at which he summarized the military applications of electroplating in World War II and participated in discussions of this and other problems. At the same conference, Dr. G. W. Vinal, chief of the Electrochemistry Section (Electricity Division), presided at a symposium on batteries, in which the Bureau's work on new primary batteries was explained. R. S. Dill, chief of the Heat Transfer Section, participated in conferences at Montreal on heating, air conditioning, and related subjects.

Dr. J. H. Dellinger, chief of the Central Radio Propagation Laboratory, served as a member of the United States delegation to a Telecommunications Conference in Moscow which took place September 28 to October 21. The Conference, a meeting of the five powers (USSR,

Great Britain, France, China, United States) to consider international radio, telegraph, and telephone regulations, laid the groundwork for a world telecommunications conference of all nations to be held in the United States next year. The work of the Central Radio Propagation Laboratory on propagation and on standard frequency dissemination constitutes basic services very useful to such conferences and to the nations in administering the international regulations.

Dr. F. G. Brickwedde attended an international conference on low temperature physics and on the physics of fundamental particles of atomic nuclei at the University of Cambridge in July, and visited the low temperature laboratories at the Universities of Cambridge, Oxford, and Leiden (Holland). Dr. Brickwedde brought back important information for the Bureau's program in the low temperature field. At the National Physical Laboratory in England Dr. Brickwedde discussed problems relating to the International Temperature Scale, which in the United States is maintained by the Bureau.

The Bureau scientists report that European science is recovering rapidly from the war; that National policies for the encouragement of science are being widely discussed, and that military restrictions on interchange of purely scientific data are being removed. There seems to be a sincere desire to promote international goodwill on the part of scientists, and there is a great desire both to learn of the progress of science in other countries and to demonstrate as a matter of national pride accomplishments in science made during the war.

More and More Fertilizer.—Farmers used greater quantities of fertilizer during the war than ever before, but not more than half as much as it would pay them to use "under conditions of national prosperity." This is one of the important conclusions of State Production Adjustment Committees, as presented in a report released recently by the U. S. Department of Agriculture.

Published as "Cropland Use and Soil Fertility Practice in War and Peace," a section on fertilizer practices points to the sharp contrast between fertilizer use in World War I and in World War II. In the first period fertilizer consumption fell materially below the pre-war period. This was in part because of the cutting off of potash imports that had come from Germany. In World War II potash was supplied from domestic sources as well as substantial supplies of synthetic nitrogen.

Measured in terms of the principal active elements and compounds necessary for plant growth, the U. S. fertilizer consumption was about 1.8 times as great in 1944 as the average in the five years before the war, says the Bureau of Agricultural Economics. The estimates of the State Production Adjustment Committees indicate that for prosperous periods the most economic supply would be nearly four times the prewar consumption and slightly more than double the 1944 consumption.

The Corn Belt States have used relatively small quantities of fertilizer, but increased their use during the war. If Corn Belt farmers followed the suggestions of the State committees, that area would make the largest increase in use of fertilizer in the postwar years, and would then rank second to the Appalachian States with the Southeastern States ranking third and the Lake States fourth in fertilizer consumption. In the past the Southeast and the Appalachian States have been close together in first and second place with the Northeast third and the Corn Belt a poor fourth.

R. H. O.

More Oils But Less Animal Fats.—More vegetable and marine oil, and somewhat smaller supplies of animal fats are in prospect for the world at large and also for the United States according to a survey of world food supplies, made when crop prospects in the northern hemisphere were fairly well evident, and published by the U. S. Department of Agriculture.

The Office of Foreign Agricultural Relations reports that the Philippine copra industry is moving along at a fast rate and that exports are picking up rapidly. The recent agreement of the Netherlands East Indies to sell its coconut oil to the United States indicates an increase in supplies from this important producing area.

Supplies of sunflower seed oil in Argentina are larger than a year ago and larger olive oil crops are in prospect in Europe.

Manchuria's soybean prospects are reported favorable, and production may run as high as 85 per cent. of average. There are some carryover stocks, but it may prove difficult to move these to seaboard, and there are exchange difficulties and political uncertainties.

World butter production has declined with the expansion and consumption of other dairy products, the survey indicates. Lard output will be reduced as a result of fewer hogs in Canada, the United States and Argentina.

R. H. O.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, LECTURE HALL 8:15, WEDNESDAY, DECEMBER 18.

The stated monthly meeting of The Franklin Institute was held at 8:15 on Wednesday, December 18, 1946, Mr. Richard T. Nalle, President, presided. The President announced that the minutes of the stated monthly meeting for October were printed in full in the November issue of the JOURNAL OF THE FRANKLIN INSTITUTE, and that if there were no corrections or additions, the minutes would be approved as published. There was no dissent.

The President then called on the Secretary for his report and announcements. The Secretary announced that the following members were elected during the month of November:

Active	34
Associate	34
Student	27
Total membership as of November 30	5545

The Secretary then announced that all seats were taken for the Christmas Week Lectures for Young People to be given under the James Mapes Dodge Foundation on December 26 and 27, and that the speaker this year was Dr. David Ludlum, whose subject was "You and the Weather."

The Secretary announced the following nominations for President, Vice-Presidents, Treasurer, and members of the Board of Managers to be made in accordance with the By-Laws at the Stated Meeting in the month of December.

President	RICHARD T. NALLE
(Serve one year)	
Vice-Presidents	WALTON FORSTALL
(Serve one year)	W. CHATTIN WETHERILL
	S. S. FELS
	RICHARD W. LLOYD
Treasurer	M. M. PRICE
(Serve one year)	
Managers	G. H. CLAMER
(Serve three years)	CLARENCE L. JORDAN
	RALPH KELLY
	LIONEL F. LEVY
	CHARLES PENROSE
	JAMES H. ROBINS
	PHILIP C. STAPLES
	PHILIP H. WARD, JR.

The President then for nominations from the floor, and since there were none, the nominations were closed.

The Secretary then announced that the Board of Managers of The Franklin Institute had nominated Dr. Vannevar Bush for election to honorary membership in The Franklin Institute. All voting Members of the Institute were asked to vote pro or con. The vote was unanimously affirmative and Dr. Bush was declared elected.

The President next presented the speaker of the evening, Mr. H. W. Field, Assistant Manager, Research and Development Department, The Atlantic Refining Company, who lectured on "New Products of the Petroleum Industry," dealing with the nature of wartime

developments in this industry and how they have been transposed with rapidity and smoothness into peacetime products. He discussed the rapid swing from combat grade aviation gasoline to superior motor fuel, and from synthetic rubber to industrial chemicals, also how the all-purpose G. I. Soap was reconverted into industrial detergents. Mr. Field showed a number of charts and diagrams.

The audience was then given an opportunity to question Mr. Fields on any points of his talk. This discussion proved very edifying. The meeting adjourned with a rising vote of thanks to the speaker.

HENRY BUTLER ALLEN, *Secretary.*

NEW MEMBERS OF THE FRANKLIN INSTITUTE ELECTED, JUNE 1, 1946, TO DECEMBER 1, 1946.

ACTIVE MEMBERS WITH FAMILY PRIVILEGES.

Mr. M. D. Abuhove
Major General G. M. Barnes
Mr. Joseph W. Berenato
Mr. S. P. Blakeman
Mr. Nathaniel Cooper
Mr. John Deal
Mr. Guy Di Ambrosio
Mr. Franz Duerr
Mr. David Furman
Mr. William H. Furry
Mr. Walter A. Harris
Mr. Harry A. Hartranft
Mr. Borden O. Hoffman
George L. Hoffman, Jr., M.D.
Mr. David A. Ingber
Mr. Benjamin Jacobs
Mr. Jacob Knup, Jr.
Mr. R. P. Lepleiter
Mr. Albert J. Miller
Mr. Eurene F. Moore, II
Mr. Edward M. Nusbickel
Mr. Frank V. Radomski
Mr. Donald H. Ross
Mr. Emanuel Sbar
Mr. S. Ralph Schwarzschild
George D. Shoup, M.D.
Mr. Harry A. Sovel
Mr. Donald M. Steele
Mr. Harold C. Stott
Mr. Louis Travaglini
Mr. Oliver S. Twist
Mr. Edward C. Van Orsdell
Mr. James L. Williams
Mr. Fred Wimmersberger
Mr. Ambrose J. Winder
Mr. Joseph A. Wintz, Jr.

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Mr. Theodore A. Andrews
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Mr. James Armour
Mr. T. E. Arnold
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Mr. Thomas J. Bagley
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Mr. Harry Bankle
Mr. Emanuel W. Beloff
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Mr. Irwin H. English
Mr. Clarence R. Eppinger
Mr. Benjamin Erisman
Mr. Benjamin Rice Faunce
Mr. James F. Fee
Mr. A. Fred Forster
Mr. Warren H. Foster
Mr. William T. Freeland
Mr. P. S. Frieder
Mr. John W. Furlow

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Mr. Gerald A. Gallagher
Mr. William F. Ganter
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Mr. George S. Geary
Mr. Carl C. Geissler
Mr. Sydney Genske
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Mr. Frank John Gozlow
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Mr. Joseph H. Wolfhart
Mr. J. H. Wood
Mr. David U. Wright, Jr.
Mr. Thomas P. Wright

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Mr. Reginald F. Chase
Mr. John F. Glenn
Mr. Björn Holmström
Mr. Arthur L. Johnson
David M. Ludlum, Ph.D.
Mr. Roy E. Murphy, Jr.
Mr. James B. Rothschild

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COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, December 11, 1946.)

HALL OF THE COMMITTEE,
PHILADELPHIA, DECEMBER 11, 1946.

DR. HIRAM S. LUKENS in the Chair.

The following reports were presented for final action:

No. 3170 } Henderson Medal.
No. 3171 }

This report recommended the award of two Henderson Medals; one to Lars Olai Grondahl, of Pittsburgh, Pennsylvania, "In consideration of his contributions over a period of years to the development which has resulted in making available a practical system of inductive train communication"; and one to

Sedgwick North Wight, of Rochester, New York, "In consideration of his accomplishments in the invention and development of major railroad signaling systems thus contributing to the safety, speed and reliability of railroad operation."

No. 3153: Work of K. S. M. Davidson.

This report recommended the award of a John Price Wetherill Medal to Kenneth Seymour Moorhead Davidson, of Hoboken, New Jersey, "In consideration of his contribution to the improved design of ships through his experimental work on ship hull models."

No. 3154: Work of Robert H. Kent.

This report recommended the award of a Howard N. Potts Medal to Robert Harrington Kent, of Aberdeen Proving Ground, Maryland, "In consideration of his outstanding contributions to the science of ballistics and particularly to his practical adaptation of scientific methods to the improvement of ordnance; for his development to a state of usefulness to science of the Aberdeen supersonic wind tunnel; and for his contributions in ballistics and aerodynamics."

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9×14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

The Blood Picture of Normal Laboratory Animals. A Review of the Literature, 1936–1946.—MARY VIRGINIA GARDNER. The literature prior to 1926 on blood studies of normal animals was reviewed by Scarborough (1). The development of new and more accurate methods for the determination and enumeration of the blood elements suggested the importance of reviewing the literature for the past decade. This review lays no claim to being exhaustive, but it brings up to date the standards of comparison for hematological work on laboratory animals which Scarborough so effectively provided twenty years ago.

Most of the values recorded here were those obtained from normal animals used as controls in the experimental work reported by the authors. For that reason the number of animals recorded is often small and sometimes the range of normal values is given rather than the mean. The species or strain used has been indicated where possible.

THE RAT.

Method of Obtaining Blood.—Blood is usually obtained from the tail, either by clipping the end of the tail or incising the vein, or by cardiac puncture.

Red Blood Cells.—The red blood cell of the rat is a biconcave disc which varies little in shape. In Wright-stained preparations the eryth-

TABLE I.

Reticulocytes.

Age	No. and Sex of Rats	Strain	Reticulocytes Per Cent. of Erythrocytes		Reported by
			average	*range*	
new-born	—	Wistar	95.0		Creskoff (3)
new-born	—	—	71.7		McCord (2)
1 day	12	U. of Chicago	88.8		Bruner (4)
Average			85.16		
4 days	12	U. of Chicago	30.1		Bruner (4)
5–25 days	12 per day	U. of Chicago		19–32	Bruner (4)
3–4 months	62 F	—	1.72		Crafts (5)
adult	21 F	U. of Chicago	2.6		Bruner (4)
adult	28 M	Wistar	4.3		Creskoff (3)
adult	22 F	Wistar	3.3		Creskoff (3)
adult	12	Conn. Ag. station	2.3		Smith, P. K. (6)
adult	12	—	3.4		Smith, S. E. (7)
adult	38	—		0.6–4.8	Vollmer (8)
Average			2.94		

rocyte is pale orange, although many erythrocytes of young animals and occasionally even erythrocytes of older ones may be slightly basophilic. With cresyl blue staining as many as 4.8 per cent. of the red blood cells may show basophilic aggregation or reticulum (Table I). Most of the erythrocytes of the new-born rat are reticulated, and McCord and Bradley (2) have found that this reticulation may persist until the 100th day of postnatal life.

The average number of red blood cells in the adult rat, regardless of sex or strain, is 8,520,000 per cmm. (Table II). The count of the new-born rat averages 2,630,000 red blood cells per cmm. In very young rats the erythrocytes are fewer in number but larger in size than in adult rats. The average mean diameter of the red blood cell of new-born or very young rats is 9.26 μ, while that of the adult is 6.34 μ.

It was found by Bruner, van de Erve and Carlson (4) that the red blood cells increase at an approximate rate of 100,000 per day for the

TABLE II.

Erythrocytes.

Age	No. and Sex of Rats	Strain	Erythrocytes $\times 10^6$/cmm.		Diameter Erythrocytes μ	Reported by
			average	*range*		
new-born	—	Wistar	—	2.5–3.5	9	Creskoff (3)
new-born	144	—	2.89		8.42	Noble (9)
new-born	92	—	2.93		9.73	Schlicke (10)
2 hours	90	Wistar × Chicago alley rat	2.68		9.08 8.23–11.72	Wigodsky (11)
1 day	12	U. of Chicago	2.4			Bruner (4)
1–8 days	162 M+F	Fischer+A X C	2.0			Reich (12)
baby	20	—	2.89	2.0–3.42	10.09 9.67–10.79	Stasney (13)
Average			2.63		9.26	
24 days	12	U. of Chicago	5.04			Bruner (4)
adult	21 F	U. of Chicago	9.422			Bruner (4)
adult	62 F	—	8.26	7.5–8.9		Crafts (5)
3–4 months	21 F	Long-Evans	8.39			Crafts (14)
adult	28 M	Wistar	9.27		6.3	Creskoff (3)
adult	22 F	Wistar	9.42		6.3	Creskoff (3)
adult	6	—	9.06	8.5–10.0		Cress (15)
adult	8 M	hooded	7.47	5.0–9.95		Dalton (16)
adult	27	Sprague-Dawley	8.88			Dougherty (17)
adult	24	Wistar	8.1735			Farris (18)
adult	15 M	Wistar	9.625	8.4–11.1		Higgins (19)
adult	4	Sherman	9.7			Kessler (20)
2½–3 months	28	some Wistar		7.0–9.0		Lawrence (21)
adult	6 M	Wistar		10.77–11.16		Machella (22)
adult	M	Wistar		9.82–10.25		Machella (23)
adult	12 M	Conn. Ag. station		6.5–9.2		Orten (24)
adult	66 M	August	8.8			Reich (12)
adult	54 F	August	8.5			Reich (12)
adult	399 M	Fischer	9.2			Reich (12)

TABLE II (*Continued*).

Age	No. and Sex of Rats	Strain	Erythrocytes $\times 10^6$/cmm.		Diameter Erythrocytes μ	Reported by
			average	*range*		
adult	320 F	Fischer	8.9			Reich (12)
adult	211 M	Sherman	8.8			Reich (12)
adult	172 F	Sherman	8.5			Reich (12)
adult	111 M	Marshall	8.9			Reich (12)
adult	139 F	Marshall	8.6			Reich (12)
adult	114 M	A X C	8.5			Reich (12)
adult	114 F	A X C	8.4			Reich (12)
adult	156 M	Silver grey	8.6			Reich (12)
adult	139 F	Silver grey	8.4			Reich (12)
adult	135 M	Zimmerman	8.8			Reich (12)
adult	152 F	Zimmerman	8.2			Reich (12)
adult	87 M	Copenhagen	8.5			Reich (12)
adult	87 F	Copenhagen	8.1			Reich (12)
adult	87 M	miscellaneous	9.3			Reich (12)
adult	113 F	miscellaneous	8.9			Reich (12)
adult	8	—	7.916			Robinson (25)
adult	38	Conn. Ag. station	7.83			Smith, P. K. (6)
adult	12	—	7.421			Smith, S. E. (7)
4–5 months	4 M	Sprague-Dawley			6.45	Stanley (26)
8 months	21 M	Rockland Farms		5.05–9.31		Vogel (27)
adult	38	—	8.35	7.9–9.1		Vollmer (8)
adult	25 M	—	8.6	7.9–9.6		Vollmer (28)
adult	50 F	—	8.2	7.4–9.1		Vollmer (28)
adult	12	U. of Denver	8.435	7.85–9.1		Wetzig (29)
adult	—	Wistar × Chicago alley rat	9.0		6.29	Wigodsky (11)
6–12 months	47 M	—	6.61			Wintrobe (30)
6–12 months	26 F	—	6.45			Wintrobe (30)
Average			8.523		6.34	

first 3 or 4 weeks of life. They found poikilocytosis and polychromatophilia only in rats less than 6 days old. Endicott and Ott (31) found 95 nucleated red blood cells per cmm. in rats less than 60 days old, but these and polychromatophilic erythrocytes disappeared when the rats were between 1 and 2 months old. The blood picture reaches the normal adult level during the 3rd or 4th month of age.

Creskoff, Fitz-Hugh and Farris (3) found that male rats average 150,000 fewer erythrocytes per cmm. of blood than do females of the same age and strain. Reich and Dunning (12), Vollmer and Gordon (28) and Wintrobe, Shumacker and Schmidt (30), however, report higher values for males than for females.

Hemoglobin.—The hemoglobin content of the blood of the adult rat averages 14.44 grams per 100 cc. (Table III). The blood of new-born rats contains, on the average, 11.86 grams of hemoglobin per 100 cc. Creskoff, Fitz-Hugh and Farris (3) and Wintrobe, Shumacker and Schmidt (30) report a higher hemoglobin content for the blood of the

TABLE III.

Hemoglobin.

Age	No. and Sex of Rats	Strain	Hemoglobin g./100 cc.		Method	Reported by
			average	*range*		
new-born	12	U. of Chicago	10.31		Newcomer	Bruner (4)
new-born	—	Wistar	11.82		Sahli	Creskoff (3)
new-born	144	—	14.11		Newcomer	Noble (9)
new-born	90	Wistar×Chicago alley rat	13.44		Newcomer	Wigodsky (11)
1–8 days	163 M+F	Fischer+A X C	9.6		Sahli	Reich (12)
Average			11.86			
17 days	12	U. of Chicago	7.14		Newcomer	Bruner (4)
24 days	12	U. of Chicago	8.52		Newcomer	Bruner (4)
young	34 M	Sherman	11.7		Klett-Summerson*	Metcoff (32)
pubescent	9 M	Sherman	12.2		Klett-Summerson*	Metcoff (32)
adult	23 M	Sprague-Dawley	15.7		Dick-Stevens*	Benditt (33)
adult	21 F	U. of Chicago	14.9		Newcomer	Bruner (4)
adult	62 F	—	15.1		Hellige	Crafts (5)
adult	28 M	Wistar	15.6		Sahli	Creskoff (3)
adult	22 F	Wistar	16.23		Sahli	Creskoff (3)
adult	27	Sprague-Dawley	14.8		Evelyn*	Dougherty (17)
adult	22 M	Sherman	15.1		Klett-Summerson*	Metcoff (32)
adult	12 M	Conn. Ag. station		10.8–16.2	acid hematin	Orten (24)
adult	66 M	August	14.8		Sahli	Reich (12)
adult	54 F	August	14.7		Sahli	Reich (12)
adult	399 M	Fischer	14.7		Sahli	Reich (12)
adult	320 F	Fischer	14.4		Sahli	Reich (12)
adult	211 M	Sherman	14.5		Sahli	Reich (12)
adult	172 F	Sherman	14.6		Sahli	Reich (12)
adult	111 M	Marshall	14.3		Sahli	Reich (12)
adult	139 F	Marshall	14.4		Sahli	Reich (12)
adult	114 M	A X C	13.9		Sahli	Reich (12)
adult	114 F	A X C	14.2		Sahli	Reich (12)
adult	156 M	Silver grey	14.2		Sahli	Reich (12)
adult	139 F	Silver grey	13.8		Sahli	Reich (12)
adult	135 M	Zimmerman	13.8		Sahli	Reich (12)
adult	152 F	Zimmerman	13.2		Sahli	Reich (12)
adult	87 M	Copenhagen	13.1		Sahli	Reich (12)
adult	87 F	Copenhagen	12.6		Sahli	Reich (12)
adult	87 M	miscellaneous	14.9		Sahli	Reich (12)
adult	113 F	miscellaneous	14.6		Sahli	Reich (12)
adult	38	Conn. Ag. station	14.43		Newcomer	Smith, P. K. (6)
adult	12	—	14.91		Beckman	Smith, S. E. (7)
8 months	21 M	Rockland Farms		13.5–17.5	—	Vogel (27)
adult	—	Wistar×Chicago	14.5		Newcomer	Wigodsky (11)
6–12 months	47 M	alley rat	13.0		Newcomer	Wintrobe (30)
6–12 months	26 F	—	13.3		Newcomer	Wintrobe (30)
Average		—	14.44			

* photoelectric colorimeter.

female than for the male, while Reich and Dunning (12) found a lower hemoglobin value in the female. Reich and Dunning also found the hemoglobin content of female rat blood to be lowered during pregnancy and lactation.

White Blood Cells.—The leukocytes of rat blood are similar morphologically to those found in human blood. The granulocytes have a coiled and twisted nucleus which may be lobulated. The cytoplasm stains palely with Wright's stain. The granules of the neutrophilic granulocytes are few, but they stain characteristically, as do those of the eosinophilic and basophilic granulocytes. The lymphocytes may be large or very small, with little or no blue-staining cytoplasm visible. The cytoplasm contains occasional azurophilic granules. The nucleus has coarsely clumped chromatin which stains deep purple. A central nucleolus is usually visible in the nucleus of the large lymphocytes. The monocyte is the largest cell to be found in the blood of the normal rat. It has a kidney-shaped nucleus surrounded by abundant cytoplasm in which there may be a few azurophilic granules.

The average total leukocyte count for the adult rat, regardless of sex or strain, is 15,050 cells per cmm. (Table IV). The average values for the various white cells, expressed as per cent. of the total white blood cell count, are:

stab neutrophils	3.85	(0–7)
segmented neutrophils	25.7	(12–39)
eosinophils	2.08	(0–3.4)
basophils	0.09	(0–1)
lymphocytes	66.97	(53–83)
monocytes	4.92	(0.7–7)

Although Creskoff, Fitz-Hugh and Farris (3) found no significant difference in the total leukocyte count of males and females, Reich and Dunning (12) believe that female rats average 820 more white cells per cmm. than do male rats. Farris (38) found that there was a reduction of 39 per cent. in the number of white cells in the blood of female rats during estrus. Farris has also shown (18) that an emotional stimulation of 5 minutes duration results in a 12.08 per cent. increase in the absolute lymphocyte count and a marked relative lymphocytosis which is maintained for at least an hour. The total leukocyte count is slightly decreased after 5 minutes stimulation. Crafts (14) found no significant change in either the total red blood cell or white cell counts under ether anesthesia.

New-born rats average 5,225 white blood cells per cmm. Endicott and Ott (31) found that the white blood cell count rose during the 1st or 2nd month after birth.

Platelets.—The platelets, or thrombocytes, are very small, blue-staining bodies, with a deeper blue or purple inclusion body. They are very numerous in stained preparations, usually appearing in heavy

TABLE IV.
Leukocytes.

Age	No. and Sex of Rats	Strain	Leukocytes per cmm.		Cells per 100 Leukocytes							Reported by
			average	*range*	Stab	Seg	Eo	Baso	Lymph	Mono	Other	
new-born	12	U. of Chicago	4,650									Bruner (4)
1–8 days	163 M+F	Fischer+A X C	5,800		2	25	0.01	0.01	49	23		Reich (12)
Average			5,225									
24 days	12	U. of Chicago	5,450									Bruner (4)
26 days	6 M	Long-Evans	6,875			24			76			Reinhardt (34)
weanling	19	Sprague-Dawley	18,200			13.7						Axelrod (35)
60 days or less	11	Wistar, New Haven or Osborne-Mendel	6,120			25.8	0.5		73.8			Endicott (31)
adult	21 F	U. of Chicago	13,476									Bruner (4)
3–4 months	21 F	Long-Evans	6,670									Crafts (14)
3–4 months	62 F	—	8,885	5,200–13,000		17.0	2.6	0.1	77.8	0.7		Crafts (5)
adult	M+F	Wistar	9,000		7	13	2	1	78	1		Creskoff (3)
adult	10	—	17,815			28.4			71.6			Cress (15)
adult	23	Osborne-Mendel or Wistar	14,469			14.9						Daft (36)
adult	8 M	hooded	16,687			12			81			Dalton (16)
adult	42	Sprague-Dawley	17,826			16			83			Dougherty (17)
adult	22 M	Wistar, New Haven or Osborne-Mendel	14,750			21	1.6		73			Endicott (37)
adult	17 F	Wistar	9,723	6,800–15,700								Farris (38)
adult	24	Wistar	14,110			35.5	2.2	0.17	60.11	1.83		Farris (18)
adult	15 M	Wistar	17,850	10,300–24,800								Higgins (19)
adult	28 M+F	some Wistar		10,000–15,000		15–20			75–80			Lawrence (21)

TABLE IV (*Continued*).

Age	No. and Sex of Rats	Strain	Leukocytes per cmm.		Cells per 100 Leukocytes							Reported by
			average	*range*	Stab	Seg	Eo	Baso	Lymph	Mono	Other	
adult	66 M	August	14,100			29	1.4	0.03	65	4		Reich (12)
adult	54 F	August	15,200			34	1.3	0.06	60	5		Reich (12)
adult	399 M	Fischer	14,000			25	1.9	0.01	66	7		Reich (12)
adult	320 F	Fischer	14,000			22	2.2	0.01	69	7		Reich (12)
adult	211 M	Sherman	14,300			28	1.9	0.00	64	6		Reich (12)
adult	172 F	Sherman	14,500			24	2.2	0.00	68	6		Reich (12)
adult	111 M	Marshall	17,500			27	2.0	0.01	65	6		Reich (12)
adult	139 F	Marshall	16,800			26	2.7	0.01	66	6		Reich (12)
adult	114 M	A X C	17,000			36	3.0	0.01	56	5		Reich (12)
adult	114 F	A X C	18,000			33	3.4	0.01	59	5		Reich (12)
adult	156 M	Silver grey	14,600			29	1.8	0.03	64	5		Reich (12)
adult	139 F	Silver grey	15,500			25	1.7	0.00	67	6		Reich (12)
adult	135 M	Zimmerman	15,300			27	1.6	0.01	65	6		Reich (12)
adult	152 F	Zimmerman	15,800			25	2.1	0.00	67	7		Reich (12)
adult	87 M	Copenhagen	23,300			39	2.0	0.00	53	6		Reich (12)
adult	87 F	Copenhagen	26,900			38	3.4	0.00	53	6		Reich (12)
adult	87 M	miscellaneous	14,000			30	2.7	0.00	63	5		Reich (12)
adult	113 F	miscellaneous	16,300			30	2.6	0.00	62	5		Reich (12)
adult	8	—	14,493		0.7	20.9	0.9	0.2	72.7	1.6	3.2	Robinson (2)
adult	38	Conn. Ag. station	9,300			15	0.7	0.4	79	5		Smith, P. K. (6)
adult	53 M	Rockland Farms		6,400–26,250		6–57	0–7		38–88	0–7		Vogel (27)
adult	100	—		14,000–24,000								Vollmer (28)
Average			15,067			25.7	2.08	0.09	66.97	4.92		

clumps. Olson (39) found the normal for 6 rats to average 754,000 per cmm., with a range of 702,000 to 796,000. Machella and Higgins (23) give, as the value for their control rats, 823,120 platelets per cmm. Creskoff, Fitz-Hugh and Farris (3) found a variation of 500,000 to 1,000,000 platelets per cmm., with an average of 800,000.

Coagulation Time.—Creskoff, Fitz-Hugh and Farris (3) and Kessler and Zwemer (20) give the normal coagulation time in the adult rat as 2.5 minutes. Ingle and Corwin (40) agree fairly closely with this estimation, giving 2.21 minutes as the normal coagulation time. Ingle and Corwin (40) found the coagulation time to be decreased after excitement of 20 minutes duration and after the administration of ether for 20 minutes.

Hematocrit.—The average normal volume of packed red cells from

TABLE V.

Hematocrit and Blood Volume.

Age	No. and Sex of Rats	Strain	Hematocrit per cent.		Blood Volume cc./100 g. wt. (method)	Reported by
			average	*range*		
new-born	92	—	45.1			Schlicke (10)
baby	20	—	43.7	39.0–53.0		Stasney (13)
new-born	90	Wistar × Chicago alley rat	38.3			Wigodsky (11)
Average			42.4			
less than 2 months	11	Wistar, New Haven, Osborne-Mendel	45.1			Endicott (31)
young	34 M	Sherman	34.7		8.9 (T 1824)	Metcoff (32)
pubescent	9 M	Sherman	36.0		10.8 (T 1824)	Metcoff (32)
adult	23 M	Sprague-Dawley	45.8		6.17 (T 1824)	Benditt (33)
adult	28 M	Wistar	49			Creskoff (3)
adult	22 F	Wistar	52			Creskoff (3)
adult	50 M+F	Wistar			6.7 (Cartland and Koch)	Creskoff (3)
adult	23	Wistar or Osborne-Mendel	44.39	35.5–51.4		Daft (36)
adult	22 M	Wistar, New Haven, Osborne-Mendel	45.6			Endicott (31)
adult	20	Albino			4.3 (Griffith and Campbell)	Griffith (41)
adult	10	Long-Evans or U. Southern Cal.	44		8.9 (T 1824)	Hechter (42)
adult	4	Sherman	46			Kessler (20)
adult	22 M	Sherman	44.6		7.22 (T 1824)	Metcoff (32)
adult	12	—	44.71			Smith, S. E. (7)
4–5 months	4 M	Sprague-Dawley	46.1		8.28 (T 1824)	Stanley (26)
6–12 months	47 M	—	38.0			Wintrobe (30)
6–12 months	26 F	—	40.4			Wintrobe (30)
Average			45.05		6.93	

the blood of the adult rat is 45.05 per cent. Both Creskoff, Fitz-Hugh and Farris (3) and Wintrobe, Shumacker and Schmidt (30) found that females have 2–3 per cent. higher hematocrit values than do males (Table V). Endicott and Ott (31) and Schlicke (10) found a value of 45.1 per cent. for new-born rats, while Stasney, Higgins and Mann (13) give 43.7 per cent. as the normal value for new-born animals and Wigodsky and Ivy (11) believe that 38.3 per cent. represents the correct value.

Specific Gravity.—The specific gravity of whole rat blood was found by Creskoff, Fitz-Hugh and Farris (3) to be 1.056, and by Dougherty and White (17) to be 1.059.

Blood Volume.—The average blood volume of the adult rat amounts to 6.93 cc./100 g. body weight. Small animals have a greater blood volume per gram of body weight than do large animals (Table V).

Resistance of Red Blood Cells.—Creskoff, Fitz-Hugh and Farris (3) report beginning hemolysis with 0.475 per cent. NaCl and complete hemolysis with 0.300 per cent. NaCl.

Sedimentation Rate.—Creskoff, Fitz-Hugh and Farris report (3) the one-hour blood sedimentation rate as 1.8 mm. for female and 0.7 mm. for male rats.

The average values recorded here for the size, number and resistance of the red blood cells, the differential leukocyte and platelet counts, the blood volume and the specific gravity agree within the limits of experimental error with those given by Scarborough (1). The average total leukocyte count was found to be somewhat higher, and the coagulation time lower, than those recorded by Scarborough. Additional values, for which no references were found by Scarborough, are given for the number of reticulocytes, the volume of packed red cells and the sedimentation rate.

(*To be continued*)

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BOOK REVIEWS.

PERIODICA TECHNICA ABBREVIATA, A List of Initial Abbreviations of Technical and Scientific Periodicals, compiled by Ake Davidsson. 52 pages, 18 × 25 cms. Stockholm, Tekniska Litteratursallskapets, 1946. Price, 7 kronor.

One of the problems common to librarians and research workers is the identification of periodical abbreviations. With the increasing use of initial letters, abbreviations have frequently become entirely meaningless unless one has a key to their significance. Mr. Davidsson a Swedish librarian, found this problem occurring constantly in his work and as a result compiled this list of initial abbreviations of scientific and technical periodicals. It includes nearly 2100 abbreviations representing not only periodicals but also other serial publications, issued by institutions, societies and commercial firms. Russian abbreviations are given in a special section at the end. The information in the list includes the abbreviation, the titles are given in a special section at the end. The information in the list includes the abbreviation, the title, and the name of the country where published. American users should note the differences, in Swedish alphabetizing, principally, in this instance, that Ö follows Z.

The usefulness of such a list is evidenced by the frequency with which the same abbreviation represents different publications. E is used for five periodicals; ChA for 3; E E for 3; R A for five, and there are any number of instances where the same abbreviation is used for two different publications. Although under such circumstances it is not always certain that one secures the correct periodical the first time, the search for the reference will be facilitated by having available such a list as the present one, which indicates what the possible publications may be to which reference is made. This list commends itself to the librarian and research worker and should be in every library attempting to do reference work with periodical literature.

GEORGE E. PETTENGILL.

SEQUENTIAL ANALYSIS OF STATISTICAL DATA: APPLICATIONS. Prepared by the Statistical Research Group, Columbia University. 6 sections, 26 × 29 cms. New York, Columbia University Press, 1945. Price $6.25.

This analysis was used during the war in sampling inspection of mass produced items to determine whether they met specification requirements. As its name implies it is a method of analyzing data while they are being collected. Restricted during the war it is now available to industry generally in the form of this book which consists of six pamphlets in a ring binder. The introduction pamphlet discusses sampling in inspection, two kinds of errors, experimentation, etc., and succeeding pamphlets cover sequential analysis when the result of a single observation is a classification as good or bad and when the result of the test is acceptance or rejection (binomial distribution), when the result of the test is a decision between two methods or products (double dichotomies), when the quality being tested is measured and when the question is whether a standard is exceeded or fallen short of (mean of normal distribution, known standard deviation, one-sided alternative), when the question is whether a lot differs from a standard (mean of normal distribution, known standard deviation, two-sided alternative), and sequential analysis of variability of quality about the average (standard deviation of normal distribution, one-sided alternative). An appendix pamphlet is devoted to some properties and principles of sequential analysis.

In addition to its industrial applications, sequential analysis is increasingly gaining favor as a tool in research.

R. H. OPPERMANN.

METALLURGY OF QUALITY STEELS, by Charles M. Parker. 248 pages, drawings and illustrations, 16 × 23 cms. New York, Reinhold Publishing Corporation, 1946. Price $6.00.

The author of this book is Secretary of the General Technical Committee of the American Iron and Steel Institute. Much of the material in the book was used in educational lectures by him given to the New York Chapter of the American Society for Metals and the Army Air Force School for Inspectors, Eastern District. The book is not designed to be a substitute for present texts on the subject but is intended to be an introduction to them.

It opens with a discussion on the general nature of steel pointing out that the two broad classes of steel covered by this volume are carbon and alloy steels of the construction or engineering group. The concept of crystal composition of metals is brought out based on interatomic forces and the arrangement of atoms in crystals, cubic lattice forms, and atomic arrangements of the constituents of steel are given emphasis. This serves as an introduction to treatments on steel quality as related to method of manufacture, to chemical composition and control in ingot practice, and to rolling mill practice. All this shows how important a part chemical analysis plays in controlling composition and to some extent the quality of steel, and also how careful manufacturing practice strives to eliminate defects. There follows a chapter devoted to inspection procedures to determine quality of steel. The mechanical testing of steel is a subject heading relating to the finishing and treatment of steel such as thermal critical points and solid solution changes, annealing, normalizing and spheroidizing. The effects of alloying and hardenability and hardening of steel is next taken up. A few of the more important special characteristics of steel are mentioned including quenching distortion, machining distortion, low and high temperature service. Finally a general description is given of the characteristics of some of the more popular standard steel series and their most common uses. The outstanding feature of the book is clarity in presentation. The book is interesting and is of special value to those concerned with the steel industry.

R. H. OPPERMANN.

PRÉCIS DE RADIOÉLECTRICITÉ, by Edmond Divoire. 222 p. illus., diagrams. 16 × 25 cms. Paris, Masson & Cie, 1945. Price 240 francs.

For engineers, men of science and students who wish a clear concise account of radio, Edmond Divoire, a professor at the University of Brussels, has written this book. It is not a book for specialists although they may find material of interest. Nor is it a popular treatise, but rather a volume which presents the fundamental principles of transmitters and receivers and their use in radio communication. Sufficient mathematical treatment has been given without overwhelming the reader with unnecessary formulae, and numerical problems have been included to demonstrate the utility of the theory.

The author begins with a brief summary of electromagnetic theory and then discusses tubes and their function in amplification. Various problems dealing with transmission are then considered including frequency modulation and television. A chapter is devoted to antennas and another to the propagation of waves, with a discussion of the various types. Receivers are the subject of two chapters and a concluding chapter discusses briefly the present and future problems of radio.

Written during the German occupation, the book naturally has no material on the spectacular war developments. It should nonetheless prove a very satisfactory review of radio for readers with a familiarity with French.

G. E. PETTENGILL.

PUBLICATIONS RECEIVED.

Fire and the Air War, edited by Horatio Bond. 262 pages, drawings and illustrations, 6 × 23 cms. Boston, National Fire Protection Association, 1946. Price $4.00.

Handbook of Chemistry, compiled and edited by Norbert Adolph Lange. Sixth edition. 2082 pages, 13 × 20 cms. Sandusky, Handbook Publishers, Inc., 1946. Price $7.00.

The American Annual of Photography, 1947., edited by F. R. Fraprie & F. I. Jordan. Volume 61. 208 pages, illustrations, 18 × 25 cms. Boston, American Photographic Publishing Company, 1946. Price $2.00 (paper), $3.00, (cloth).

CURRENT TOPICS.

Commerical Radar on American Owned and Operated Merchant Vessel.—Initial reports on the first commercial radar installed on an American-owned and operated merchant vessel have revealed that after five months operation, it has materially helped in maintaining sailing schedules with resultant savings in time and expense, according to officials of the American South African Line.

The General Electric electronic navigator aboard the line's flagship, the *SS. African Star*, was installed and placed in operation on the ship's maiden voyage. Encountering fog and storms on runs between the United States and South Africa, the navigator helped materially to maintain the ship's sailing schedules, according to Captain C. W. Schmidt, master of the *SS. African Star*.

Captain Schmidt cited a typical case in which the electronic nagivator proved its effectiveness.

"On a voyage from Port Elizabeth to Capetown, South Africa, heavy fog was encountered before rounding the Cape of Good Hope. In spite of this, with the radar operating, a speed of about 17.5 knots was maintained, and we arrived off Capetown breakwater at 4:00 P.M., the time previously radioed to our agents. Consequently all arrangements made as to gangs, immigration, and port authorities were not upset by a late arrival, and stevedores did not have to be paid for time awaiting ship's arrival."

The electronic navigator on the *African Star* is a model MN-1A, 10-cm. surface search radar specifically designed for use on merchant ships as a surface obstacle detector and navigational aid.

Radio microwaves emanating from the rotating antenna of the navigator, traveling at the speed of light, will detect through darkness, fog, and storm, surface objects such as buoys, lighthouses, other ships, and land masses. The relative position and range of these surface obstacles are accurately plotted, electrically, on the viewing screen located in the pilot house. A picture of the ship's surroundings is quickly and easily obtained through the simple operation of a minimum number of dials and switches conveniently located on top of the console and adjacent to the viewing screen. Range selection of 2, 6, or 30 miles radius is provided so that surroundings may be viewed on an extended or small scale basis depending upon the waters being navigated.

R. H. O.

Seven Fat Years Followed Lean.—Experience in the United States has run just contrary to the Egyptian chronicle of the seven lean years following the seven fat years of the Pharaoh's dream which Joseph interpreted. Here the seven "fat" years of high production have followed instead of coming before the seven "lean" years preceding (1933–39). However, as a result of the change in demand—both national and world-wide—the farm situation in the "lean" years was characterized by a threatening and persistent "surplus" problem. In the "fat" years, the parade of record-breaking crops has not been able to match the war-created requirements, and "shortages" have accompanied bumper crops.

These contrasts appear in U. S. Department of Agriculture tabulations of crop production. By using the August Crop Report figures for the seventh fat year, and official estimates for previous years, simple addition shows a production of more than 21 billion bushels of corn for the seven years of 1940 to 1946. For the seven previous years the "lean" production was 15.4 billion bushels. For wheat, the figures show almost 7 billion bushels in the seven fat years and 4.8 billions in the lean period. For oats, the record stands at more than 9 billion bushels compared with 6.5 billion.

The record reveals that for these crops, production in two "fat" years have been nearly equal to three of the "lean" years. And the United States has experienced seven successive fat years of good crops! Crop experts agree that the weather is a principal item. They mention, as other elements in the picture, hard work and planning by farmers, improved machinery, and better cultural practices that include such items as use of fertilizer, hybrid corn and improved varieties of other crops, and control of erosion. From the standpoint of human nutrition, substitution of machines for horses as farm power has released much acreage for food growing.

R. H. O.

Long Chain Chemistry.—"Recent development in long-chain chemistry have opened new vistas to the scientific mind," says Dr. T. L. Swenson of the U. S. Department of Agriculture in discussing advances in the application of fundamental scientific theory to research on agricultural products.

Applying both chemical and physical methods, scientists at the Western Regional Research Laboratory at Albany, Calif., have learned new ways of manipulating large molecules of protein substances into much larger and longer molecules of the long-chain type. Side chains on these long molecules are then induced to make chemical bonds with other side chains. These operations resemble in many ways those that the silk worm follows in creating the silk fiber. These inter-connected long molecules may then be forced through a spinning device comparable to the silk worm's and they emerge as a fiber of the same general type of construction as the silk fiber. The silk worm stretches the natural fiber as it hardens, and scientific fiber makers have found that stretching also improves the strength and quality of synthetic fibers.

"New fiber," says Dr. Swenson, who is director of the Western Laboratory, "which may vary widely in properties and may have many useful properties, are within the range of possibility. We need only to look, for example, at Rayon and Nylon to see the possibilities that exist when appropriate means are available for the manipulation of long-chain molecules. We have Rayon, Nylon, Vinyon, and glass fibers, but each has its own characteristic properties. Similarly we have the natural fiber—each well adapted to certain uses. And we are attracted by the possibility of the discovery of artificial protein fibers that have new and special properties not found in any other fibers.

R. H. O.

Bamboo Substitutes for Dinner Bell.—In the Ecuador highlands a 5-foot trumpet made from one internode of a thin-walled native bamboo serves as a counterpart of our dinner bell or conch shell in calling farm workers to dinner or to end the day's work, Dr. F. A. McClure of the Office of Foreign Agricul-

tural Relations recounts in the October issue of *Agriculture in the Americas*. A trumpeter with a "bocina," as it is called, goes ahead when a herd of cattle is going to market, "blaring the warning to weary travellers on foot to retire to a place of safety."

The Ecuadorians, Dr. McClure finds, are ingenious in using native bamboos for construction, for furniture, and in weaving baskets and hats. Children make bamboo toys of many kinds. McClure is a consultant on bamboo and has been promoting trial plantings in this area and elsewhere in South America. Interchange of the best and most useful South American species of bamboos between the several republics would be beneficial. Dr. McClure also reports a fair prospect for introduction of timber bamboos from the Orient. Two Oriental species have reached Ecuador from California where they are also under test. Dr. McClure does not anticipate that these will grow to the size they do in Asia, but for some purposes are likely to develop into better construction material than any of the native bamboos.

R. H. O.

Germans Behind U. S. in Automatic Controls. (*Heating, Piping and Air Conditioning*, Vol. 18, No. 10.)—German use of automatic controls for measuring temperature, pressure, humidity, and other factors is about 10 to 15 years behind current practice in the United States, according to four American experts who investigated the German industrial instrument industry under the auspices of the technical industrial intelligence division of the office of technical services, U. S. Department of Commerce. Development of industrial instruments in Germany is about 5 to 10 years behind the American industry, in the investigators' opinion.

The wartime role of industrial instruments in Germany was small as compared with that in America, the investigators reported. Apparently enough low cost labor was available to permit hand operation for most industrial measuring purposes. Also, large scale use of the "batch" method of processing in Germany permits greater dependence upon hand controls than does the "continuous" method used in the United States.

The only new material found in use by the German industrial instrument industry was polyvinyl thermoplastic, manufactured by I. G. Farben under the trade name "Igelit." The material was fabricated on a large scale into vessels, pipes, fittings, cases, and other items. By using a strip of the plastic as a welding rod, the material could be welded with a hot air jet from a special torch.

A specific gravity recorder installation on ammonium nitrate solution, using the air bubble system, was another I. G. Farben development of interest to the investigators. The range was from 1.25 to 1.40 specific gravity. Attached to some of the low range balance meters were relief seals and mercury catch pots with flexible connections, so that mercury which overflowed from the ring balance could be quickly poured out in a beaker and returned to the ring.

Flow measurement, used to guide manual control and to account for materials, is considered of great importance by the Germans, the investigators reported.

Diaphragm types of pressure gages are used in greater quantities in Germany than in the United States. Carbon steel, of 0.35 or 0.95 carbon content hardened and tempered to 350–425 Brinell, is generally used to make the gages.

Vapor pressure thermal systems are not widely used by the Germans. Though liquid expansion thermal systems are known, they are used only to a limited extent.

Thermocouples and, to a lesser extent, resistance thermometers are commonly used to measure temperature in cases where glass thermometers are not used. Standard German thermocouple accuracies are extremely low, in the opinion of the investigators.

Little use is made of liquid density measurement. Gas density measurement, however, has been the subject of intensive development and is now widely used.

Inexpensive, nonindicating, nondirect set controllers are employed for pressure control to a much smaller extent in Germany than in the United States.

Pneumatic control appears to be in its infancy in Germany. An unusual floating pneumatic controller for flow and flow ratio, operated from positive meters, has been produced by Siemens and Halske. A small amount of throttling range is secured by closing the nozzle with a tapered needle instead of a flapper.

The investigators' 131 page report on the German plants and equipment inspected is available from OTS (Study of the Industrial Processing Instrument Industry in Germany; PB4600: photostat, $9; microfilm, $1.50). Orders for reports should be addressed to the Office of Technical Services, Department of Commerce, Washington 25, D. C., and should be accompanied by check or money order, payable to the Treasurer of the United States.

"Rainbow" Marks High in Large Plane Speeds. (*Product Engineering*, Vol. XVII, No. 10.)—Republic Aviation Corporation's military photo reconnaissance plane, the XF-12, was revealed recently as the fastest long range, four-engine trasport-type airplane in the world. The military prototype of the 46-passenger "Rainbow" transport plane, the XF-12 is said to mark a new era for multiple-engined passenger airlines in global transportation. Its speed is over 450 miles per hour and its service ceiling is over 44,000 feet.

The 51-ton plane, propelled by more than 12,000 horsepower and measuring 129 feet, 2 inches from wing tip to wing tip, was developed as a flying photographic laboratory for the Air Technical Service Command of the Army Air Forces.

The plane is powered by four 28-cylinder, 3,000-horsepower Pratt and Whitney engines turning 16-feet, 2-inch four-blade propellers. Engineers expect the plane to operate at very high altitudes at speeds comparable to wartime fighter planes.

Slightly smaller than the B-29, the cigar-shaped fuselage of the new plane is 93 feet, 10 inches long. Height to the top of the stabilizer is 28 feet, 4 inches. Unique in its design for specialized reconnaissance duties, the XF-12 carries complete radio and radar equipment, armor plate and flash bombs for night photography. Specifications include three camear stations—one split vertical,

one trimetrogron, plus one vertical camera station. Darkroom equipment for loading film is included.

Housed in nacelles the size of P-47 fuselages, the four engines have 2-speed cooling fans. Each engine is fitted with two General Electric superchargers. The long nacelles are tapered over exhaust turbines from which engineers hope to garner additional thrust of an estimated 100 horsepower per engine. Air intake of the plane is in the leading edge of the wings. It has heat de-icing of wings and tail.

Republic recently announced that, with changes in power plant and interior design, the XF-12 military version is actually the forerunner of the 46-passenger Republic Rainbow Transport. This commercial version, according to Republic, will be the fastest means of long range aerial transportation ever afforded mankind. Scheduled for delivery to the airlines in 1947, the Rainbow will be driven by more powerful engines of later Pratt & Whitney design developing even greater horsepower, and fuselage of the passenger liner will be approximately five feet longer than the XF-12.

R. H. OPPERMANN.